Comparing Indefinitely Renewable Copyright and the Current Copyright System in International Setting[1]

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Abstract As an alternative to the current international copyright system, indefinitely renewable copyright (IRC) has not been compared to the current system in international settings. We endeavor to compare them in a two-country setting. By developing a two-country model of IRC and comparing it with that of the current copyright system, we find that optimally configured IRC does not necessarily lead to higher national or global welfare than an optimally configured fixed length copyright (FLC) system. National and global welfare under IRC can be lower than those under FLC, if consumer preference for variety of information products is not very strong or demand for information products is elastic; under opposite conditions, welfare IRC can be higher. IRC does seem to lead to longer copyright than the current system.

Keywords: indefinitely renewable copyright, fixed length copyright, international copyright market, modeling, simulation

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1. Introduction

We model infinitely renewable copyright (IRC) in an international setting and compare it with a model of the current international copyright system. The current copyright system faces serious

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technological and theoretical challenges. The current system is a fixed length copyright (FLC) system. Under FLC, certain exclusive rights are protected for a fixed period of time for creators of information products. The length of protection is fixed by the copyright authorities of individual countries, subject to the influence from other countries. For example, copyright length in the US is currently life plus 70 years, i.e. 70 years from the death of the last author to die for works of individual or joint authorship; and that in Canada or China is life plus 50 years.

IRC was proposed as an alternative to the current fixed length copyright system by Landes and Posner (2003). Under IRC, each creator has the option to renew the copyright of their works by paying a copyright fee. Landes and Posner suggested some desirable properties of IRC, from possibly expanding the public domain, to reducing transaction costs, to avoiding rent seeking by right owners, etc. Landes and Posner's discussion was informal and based on a single economy and single copyright authority.

IRC has been modeled and compared with FLC under single country settings by Adilov (2005) and Yuan (2006). Adilov (2005) suggests that IRC and FLC can mimic each other without effecting market outcome. Yuan (2006) suggest IRC may lead to lower social welfare than FLC.

However, IRC should be studied and compared to FLC under international setting. It is apparent that any copyright system, if implemented, must be international, due to international trade of copyright works. Consumers in one country consume and benefit from works by foreign creators. Creators in different countries compete with each other in both domestic and international markets. Copyright laws of one country affect creators of this country as well as creators of other countries, and through them, domestic and foreign consumers. Therefore, copyright laws of different countries affect each other.

This paper develops a two-country model of IRC and compares it with a similar two-country model of FLC. In this model, two countries, each with a creative industry and a market for information products, trade in these products. The copyright authorities of the two countries play a two-stage game against each other in copyright fee. In the first stage, copyright authorities choose copyright fees per period per work charged to owners of copyright of information products which are sold on the markets in the countries of the respective authorities. In the second stage, creators react to the copyright fees in deciding whether to renew the copyright for their products in each of the two markets and making other creative and marketing decisions to maximize profits. The copyright authorities set copyright fees to maximize social welfare of their respective countries, considering the effect on the behavior of the creators. We compare this model of IRC to the two-country model of FLC in Yuan (2009).

The main results of the paper are: 1) that IRC does not necessarily lead to higher national or global welfare than the current copyright system. 2) Whether IRC leads to higher welfare depends on consumer preference. IRC can lead to higher welfare if consumer preference for variety of information products is strong and price elasticity of information products is high; otherwise it may lead to lower welfare. 3) IRC seems to result in longer copyright duration than FLC.

The rest of the paper is organized as follows. The next section develops the model of IRC in a two-country setting. The third section presents the results of the simulation of the model and compares them with those of the model of FLC of Yuan (2009). The paper ends with some concluding remarks.

2. A Two-country Model of Indefinitely Renewable Copyright

2.1 The Market Setup

The market setup is similar to that in (Yuan, 2009). There are two differences. First, policy variables controlled by copyright authorities is copyright fees under IRC, not copyright durations as under FLC; second, creators set durations of copyright for their information products under IRC.

The setup is as follows: There is a world information economy composed of two countries. Each country has a sector of creators and a market for information products. A creator in either country develops first-copy information products and sells copies of its products on the domestic and foreign markets.

The copyright authority of each country maximizes social welfare in its own country. It sets its copyright policy to maximize national welfare, taking the copyright policy of the other country as given. The policy adopted by a country applies to both domestic products and foreign products on the market of that country. If prices of the same products differ on the two markets or copyright protection on one market expires before on the other, an effective ban on parallel importation will be assumed.

2.2 The IRC Model

Assume the following notations:

i, j: indices of creators of country 1 or 2;
n_k: number of creators of country k=1, 2;
s_{ki}: number of first-copy products of creator i of country k, k=1, 2;
s_k: vector of number of first-copy products of creators of country k, k=1, 2;
$s_{k\text{-}i}$: vector of number of first-copy products of all creators of country 1 and 2, other than i of country k, k=1, 2;

S: total number of first-copy products $S \equiv \sum_{k=1}^{2} \sum_{i=1}^{n_k} s_{ki}$
$C_{ki}(s_{1i})$: creative cost of creator i of country k=1, 2;
b: reproduction cost per copy of creators of both country 1 and 2;
p_{kmit}: price per copy of products of creator i of country k in country m at time t, k=1, 2; m=1, 2;
$p_{km\text{-}it}$: vector of prices of products of all creators of country 1 and 2 on the market of country m, other than the price of creator i of country k on the market of country 1, at time t, k=1, 2; m=1, 2;
f_k: copyright fee per first-copy product per unit of time of copyright in country k, k=1, 2;
T_{kmi}: copyright length chosen by creator i of country k for its products in country m, k=1 ,2; m=1, 2.
$d_{kmit}(s_{ki}, s_{k\text{-}i}, p_{kmi}, p_{km\text{-}i}, t)$: rate of demand for products of creator i of country k in country m at time t, k=1, 2; m=1, 2;
W_k: consumer surplus of country k, k=1, 2;
γ: social discount rate for consumers and creators in both countries.

A creator makes revenue on both markets, pay copyright fee in both countries, and incurs creative cost. The profit of creator i of country 1 is:

$$\pi_{1i} = \int_0^{T_{11i}} [d_{11it} \times (p_{11it} - b) - f_1 \times s_{1i}] e^{-\gamma t} dt$$

$$+ \int_0^{T_{12i}} [d_{12it} \times (p_{12it} - b) - f_2 \times s_{1i}] e^{-\gamma t} dt - c_{1i}(s_{1i}) \quad (1)$$

The first term is the present value of the quasi rent from selling its products on the market of country 1 during copyright duration of T_{11i} chosen by the creator in that country, minus the copyright fees paid during the period. The second term is the present value of the quasi rent from selling on the market of country 2 during copyright duration of T_{12i} chosen by the creator in that country, minus the copyright fees paid in that country. The third term is the creative cost.

A creator maximizes profit by choosing the prices of its products, the duration of copyright for its products on the two markets, and the number of first-copy products to create. The creator also decides whether to enter or stay on the market. The creative industries are assumed to be open. Therefore, entry and exit stop when the marginal creator makes zero economic profits. If all creators in a country have the same technology, they will all make zero profit.

The behavior of creator i of country 1 can be written as:

$$\frac{\partial \pi_{1i}}{\partial p_{11it}} = \frac{\partial \pi_{1i}}{\partial p_{12it}} = \frac{\partial \pi_{1i}}{\partial T_{11i}} = \frac{\partial \pi_{1i}}{\partial T_{12i}} = \frac{\partial \pi_{1i}}{\partial s_{1i}} = \pi_{1i} = 0 \tag{2}$$

The first two terms are the first-order conditions of the price decisions; the third and fourth terms are those of the copyright renewal decisions on the two markets; the fifth term is the first-order condition of decision on the number of first-copy products to creative. The last term is the entry condition in equilibrium.

Similarly, the profit of creator i of country 2 is:

$$\pi_{2i} = \int_0^{T_{21i}} [d_{21it} \times (p_{21it} - b) - f_1 \times s_{2i}] e^{-\gamma t} dt$$

$$+ \int_0^{T_{22i}} [d_{22it} \times (p_{22it} - b) - f_2 \times s_{2i}] e^{-\gamma t} dt - c_{2i}(s_{2i}) \tag{3}$$

The behavior of creator i of country 2 can be described as:

$$\frac{\partial \pi_{2i}}{\partial p_{21it}} = \frac{\partial \pi_{2i}}{\partial p_{22it}} = \frac{\partial \pi_{2i}}{\partial T_{21i}} = \frac{\partial \pi_{2i}}{\partial T_{22i}} = \frac{\partial \pi_{2i}}{\partial s_{2i}} = \pi_{2i} = 0 \tag{4}$$

The national welfare of a country includes the consumer surplus, profits of creators of the country, and copyright fee collected by the copyright authority. The national welfare of country 1 can be written as:

$$W_1 = \sum_{i=1}^{n_1} \int_0^{\infty} \left(\int_b^{\infty} d_{11i} dp_{11it} \right) e^{-\gamma t} dt + \sum_{i=1}^{n_2} \int_0^{\infty} \left(\int_b^{\infty} d_{21i} dp_{21it} \right) e^{-\gamma t} dt$$

$$- \sum_{i=1}^{n_1} \int_0^{T_{11i}} \left(\int_b^{p_{11it}^{*}} d_{11i} dp_{11it} \right) e^{-\gamma t} dt - \sum_{i=1}^{n_2} \int_0^{T_{21i}} \left(\int_b^{p_{21it}^{*}} d_{21i} dp_{21it} \right) e^{-\gamma t} dt$$

$$+ \sum_{i=1}^{n_1} \int_0^{T_{11i}} (s_{1i} \times f_1) e^{-\gamma t} dt + \sum_{i=1}^{n_2} \int_0^{T_{21i}} (s_{2i} \times f_1) e^{-\gamma t} dt \qquad (5)$$

Where p^{*}_{11it} and p^{*}_{21it} are prices chosen by creator i of country 1 and creator i of country 2, respectively, on the market of country 1 during copyright protection. The first term is country 1's consumer surplus from all products of creators of country 1, if the products were priced at marginal reproduction cost b from the moment they are created; the second term is the surplus from products of creators of country 2, if the products were priced at reproduction cost b from the moment they are created; the third term is the loss of consumer surplus from products of creators of country 1 due to copyright protection of length T_{11i} chosen by the creators; the fourth term is the loss of consumer surplus from products of creators of country 2 due to the copyright protection of length T_{21i}. The fifth terms is the present value of the copyright fees paid by creators of country 1; the last term is the copyright fee paid by creators of country 2. Creator profits do not appear in the national welfare function, as creators make zero economic profit in equilibrium under the assumption that creators have the same technology.

The copyright authority of country 1 chooses its copyright fee to maximize national welfare, given the behavior of the creators of country 1 and 2, the country 2's copyright fee:

$$\max_{f_1} W_1$$

S.t. (2) and (4), and (8) (6)

(2) and (4) are creators' behavior; (8) is the behavior of copyright authority of country 2. The solution of (6) gives a reaction function of f_1 to f_2.

Similarly, the national welfare of country 2 is:

$$W_2 = \sum_{i=1}^{n_1} \int_0^\infty \left(\int_b^\infty d_{12i} dp_{12it} \right) e^{-\gamma t} dt + \sum_{i=1}^{n_2} \int_0^\infty \left(\int_b^\infty d_{22i} dp_{22it} \right) e^{-\gamma t} dt$$

$$- \sum_{i=1}^{n_1} \int_0^{T_{12i}} \left(\int_b^{p_{12it}^*} d_{12i} dp_{12it} \right) e^{-\gamma t} dt - \sum_{i=1}^{n_2} \int_0^{T_{22i}} \left(\int_b^{p_{22it}^*} d_{22i} dp_{22it} \right) e^{-\gamma t} dt$$

$$+ \sum_{i=1}^{n_1} \int_0^{T_{12i}} (s_{1i} \times f_2) e^{-\gamma t} dt + \sum_{i=1}^{n_2} \int_0^{T_{22i}} (s_{2i} \times f_2) e^{-\gamma t} dt \qquad (7)$$

The problem of the copyright authority of country 2 is to choose its copyright fee to maximize its own national welfare, given the behavior of the creators of country 1 and country 2 in the country, and country 1's copyright fee:

$$\max_{f_2} W_2$$

S.t. (2) and (4), and (6) (8)

The solution of (8) gives a reaction function of f_2 to f_1.

The two reaction functions form (6) and (8) determine equilibrium copyright fees of f_1 and f_2, which then determine the prices, copyright duration, number of first-copy products per creator, and number of creators.

2.3 The Fixed Length Copyright Model

The FLC model in is copied here from (Yuan, 2009) for convenience. Let T_1 be the copyright duration of country 1 and T_2 the duration in country 2, set by the copyright authorities. The profit of creator i of country 1is:

$$\pi_{1i} = \int_0^{T_1}(d_{11it}(p_{11it} - b)e^{-\gamma t}dt + \int_0^{T_2}(d_{12it}(p_{12it} - b)e^{-\gamma t}dt - c_{1i}(s_{1i}) \tag{9}$$

The profit of creator i of country 2 is:

$$\pi_{2i} = \int_0^{T_1}(d_{21it}(p_{21it} - b)e^{-\gamma t}dt + \int_0^{T_2}(d_{22it}(p_{22it} - b)e^{-\gamma t}dt - c_{2i}(s_{2i}) \tag{10}$$

A creator chooses prices and number of first-copy products and it decides whether to enter the market:

$$\frac{\partial \pi_{1i}}{\partial p_{11it}} = \frac{\partial \pi_{1i}}{\partial p_{12it}} = \frac{\partial \pi_{1i}}{\partial s_{1i}} = \pi_{1i} = 0 \tag{11}$$

and

$$\frac{\partial \pi_{2i}}{\partial p_{21it}} = \frac{\partial \pi_{2i}}{\partial p_{22it}} = \frac{\partial \pi_{2i}}{\partial s_{2i}} = \pi_{2i} = 0 \tag{12}$$

National welfare of country 1 under FLC is:

$$W_1 = \sum_{i=1}^{n_1} \int_0^{\infty} \left(\int_b^{\infty} d_{11i} dp_{11it} \right) e^{-\gamma t} dt + \sum_{i=1}^{n_2} \int_0^{\infty} \left(\int_b^{\infty} d_{21i} dp_{21it} \right) e^{-\gamma t} dt$$

$$- \sum_{i=1}^{n_1} \int_0^{T_1} \left(\int_b^{p^*_{11it}} d_{11i} dp_{11it} \right) e^{-\gamma t} dt - \sum_{i=1}^{n_2} \int_0^{T_1} \left(\int_b^{p^*_{21it}} d_{21i} dp_{21it} \right) e^{-\gamma t} dt \tag{13}$$

That of country 2 is:

$$W_2 = \sum_{i=1}^{n_1} \int_0^{\infty} \left(\int_b^{\infty} d_{12i} dp_{12it} \right) e^{-\gamma t} dt + \sum_{i=1}^{n_2} \int_0^{\infty} \left(\int_b^{\infty} d_{22i} dp_{22it} \right) e^{-\gamma t} dt$$

$$- \sum_{i=1}^{n_1} \int_0^{T_2} \left(\int_b^{p^*_{12it}} d_{12i} dp_{12it} \right) e^{-\gamma t} dt - \sum_{i=1}^{n_2} \int_0^{T_2} \left(\int_b^{p^*_{22it}} d_{22i} dp_{22it} \right) e^{-\gamma t} dt \quad (14)$$

The problem of copyright authority of country 1 is:

$$\max_{T_1} W_1$$

$$\text{S.t. (11) and (12), and (16)} \quad (15)$$

The problem of copyright authority of country 2 is:

$$\max_{T_2} W_2$$

$$\text{S.t. (11) and (12), and (15)} \quad (16)$$

3. Solution

3.1 Specification of Demand and Cost Functions

Solving the models requires specific forms of the demand and cost functions. Following (Yuan, 2009), assume the following demand functions:

$$d_{11it} = D_1 s_{1i} \left(\sum_{j=1}^{n_1} s_{1j} + \sum_{j=1}^{n_2} s_{2j} \right)^{\alpha-1} p_{11it}{}^{-\delta} \prod_{j \neq i} p_{11jt}{}^{\frac{\beta}{n_1+n_2-1}} \prod_{j=1}^{n_2} p_{21jt}{}^{\frac{\beta}{n_1+n_2-1}} g_1(t) \quad (17)$$

$$d_{12it} = D_2 s_{1i} \left(\sum_{j=1}^{n_1} s_{1j} + \sum_{j=1}^{n_2} s_{2j} \right)^{\alpha-1} p_{12it}{}^{-\delta} \prod_{j \neq i} p_{12jt}{}^{\frac{\beta}{n_1+n_2-1}} \prod_{j=1}^{n_2} p_{22jt}{}^{\frac{\beta}{n_1+n_2-1}} g_2(t) \quad (18)$$

$$d_{21it} = D_1 s_{2i} \left(\sum_{j=1}^{n_1} s_{1j} + \sum_{j=1}^{n_2} s_{2j} \right)^{\alpha-1} p_{21it}{}^{-\delta} \prod_{j \neq i} p_{21jt}{}^{\frac{\beta}{n_1+n_2-1}} \prod_{j=1}^{n_1} p_{11jt}{}^{\frac{\beta}{n_1+n_2-1}} g_1(t) \quad (19)$$

$$d_{22it} = D_2 s_{2i} \left(\sum_{j=1}^{n_1} s_{1j} + \sum_{j=1}^{n_2} s_{2j} \right)^{\alpha-1} p_{22it}{}^{-\delta} \prod_{j \neq i} p_{22jt}{}^{\frac{\beta}{n_1+n_2-1}} \prod_{j=1}^{n_1} p_{12jt}{}^{\frac{\beta}{n_1+n_2-1}} g_2(t) \quad (20)$$

where

$$g_1(t) = \begin{cases} 1 - \dfrac{t}{T_{01}} & if \quad t < T_{01}(1-\theta_1) \\ \theta_1 & otherwise \end{cases} \tag{21}$$

$$g_2(t) = \begin{cases} 1 - \dfrac{t}{T_{02}} & if \quad t < T_{02}(1-\theta_2) \\ \theta_2 & otherwise \end{cases} \tag{22}$$

And assume the cost functions:

$$c_{1i}(s_{1i}) = c_{01} + a_1 s_{1i}^{\rho_1} \qquad \forall \text{ i of country 1} \tag{23}$$

$$c_{2i}(s_{2i}) = c_{02} + a_2 s_{2i}^{\rho_2} \qquad \forall \text{ i of country 2} \tag{24}$$

where $0<\alpha<1$, $\delta>1$, $\beta>0$, $0\leq \theta_1 <1$, $0\leq \theta_2 <1$, $\rho_1>1$, $\rho_2>1$, and D_1, D_2, T_{01}, T_{02}, c_{01}, c_{02} and a_1 and a_2 are positive constants.

The main features in demand functions (17)-(20) are:

1) There are five factors which multiplicatively affect the demand for products of a creator: (i) the number of first-copy products of the creator, (ii) the total number of first-copy products on the market from all creators, (iii) the price of products of this creator, (iv) the prices of products of other creators, and (v) time.
2) The total demand for all information products of all creators on a market increases with the total number of first-copy product. The parameter α is the speed of the increase. It describes the consumers' preference for product variety. And $0<\alpha<1$ represents that the products are substitutes.

3) The total demand is distributed among creators in proportion to their numbers of first-copy products, everything else being equal.
4) The demand for the products of a creator decreases with the price charged by the creator. The parameter δ is the price elasticity. $\delta > 1$ is necessary for the consumer surpluses to be finite.
5) The demand for the products of a creator increases with the prices of other creators. The parameter $\beta > 0$ is the cross-price elasticity.
6) The demands in the two markets decrease over time to residual levels of θ_1 and θ_2 of the original demands in time $T_{01}*(1- \theta_1)$ and $T_{02}*(1- \theta_2)$, respectively.
7) How demand change over time depends on the market, not on the origin of the products.

The markets in the two countries may differ in the level of demand, D_1 and D_2, and the residual demand, θ_1 and θ_2, and the time it takes for the demands to drop to the residuals, $T_{01}(1-\theta_1)$ and $T_{02}(1-\theta_2)$. T_{01} and T_{02} will be referred to as the economic life of products on the two markets.

Otherwise, each market treats all domestic and foreign products similarly. And consumers in the two countries have the same price elasticity, cross-price elasticity, and preference for variety, as represented by the common values of δ, β, and α, respectively.

The main features of the creative cost functions of (23) and (24) are:

1) There are fixed costs to enter the creative industries in both countries, represented by c_{01} and c_{02}, respectively.
2) There are decreasing returns to scale in creation in both countries, as reflected in the parameters $\rho_1 > 1$, $\rho_2 > 1$, respectively.

3) The levels of variable creative costs also depend on the parameters a_1 and a_2, respectively, which will be referred to as the "per-product creative cost" parameters.

Creators within one country have identical creative costs. Creators of one country may differ from creators of the other country in fixed creative cost, per-product creative cost, and economies of creative scale, perhaps due to technological and general regulatory differences.

Given the multiplicativity of the factors affecting the demand, the common price elasticity, and the common reproductive cost of b for all products, it is easy to derive that creators set prices which are uniform for all products, all creators, at all moments of time:

$$p_{11it} = p_{12it} = p_{21j} = p_{22j} = p \equiv \frac{\delta}{\delta-1} b \qquad (25)$$

Given the identical cost functions within one country, it can be derived that creators of one country all create the same number of first-copy products: $s_{1i} = s_{1j} \equiv s_1$ and $s_{2i} = s_{2j} \equiv s_2$; and all creators choose the same copyright duration in a given country: $T_{11i} = T_{11j} = T_{21i} = T_{21j} \equiv T_1$, and $T_{12i} = T_{22j} = T_{22i} = T_{22j} \equiv T_2$.

We need to further solve for price p, sizes of creators, s_1 and s_2, total number of first-copy products S, copyright duration T_1 and T_2, and copyright fees f_1 and f_2. Given the above demand and cost functions, analytical solution is not found. Numerical methods are used to solve the model for given values of the parameters in the demand and cost functions.

3.2 The Result

Assume the following parameter values:

$$[D_1, D_2, \alpha, \delta, \beta, b, T_{01}, T_{02}, \theta_1, \theta_2, \gamma, c_{01}, c_{02}, a_1, a_2, \rho_1, \rho_2] =$$

$$[7*10^7\ \ 7*10^7\ 0.4\ \ 2\ \ 0.5\ \ 5\ \ 100\ 100\ \ 0.001\ 0.001\ 0.05\ 3*10^6\ 3*10^6\ 10^4\ 10^4\ 1.2\ 1.2\ ] \qquad (26)$$

These values are selected not to represent any particular market but to be within reasonable ranges and will be changed later. By the above parameter values, the creators of two countries are assumed to have the same technologies and consumers the same preferences.

With these parameter values, the solution of the IRC model, compared to that of the FLC model, is given in table 1.

Table 1: A Comparison of IRC and FLC

	IRC	**FLC**
Country 1 copyright fee: f_1 ($)	933	n/a
Country 2 copyright fee: f_2($)	933	n/a
Country 1 duration: T_1	61	11
Country 2 duration: T_2	61	11
Country 1 creator size: s_1	443	443
Country 2 creator size: s_2	443	443
Number of first-copy products: S (1000)	1,271	1,168
Number of copies sold in first 100 years (Billion)	78	145
Country 1 social welfare: W_1($B)	122	142
Country 2 social welfare: W_2($B)	122	142

Under IRC, copyright fees in the two countries are both $933 per product per year. These fees induce creators to choose 61 years of copyright protection for their products, much longer than the optimal copyright duration of 11 years chosen by copyright authorities under FLC. Creators in the two countries create 1,271,401first-copy products together under IRC, 9% more than the number under FLC. The number of first-copy product created by each creator will be similar, 433, under both IRC and FLC. Global consumption of information products in the first

100 years is 78 billion copies under IRC, 46% fewer than the 145 billion copies under FLC. Finally, social welfare of each country is $122 billion under IRC, 14% lower than the $142 billion for each country under FLC.

Figure 1 and 2 show the optimality of the IRC solution. Figure 1 shows that, first, each creator makes zero economic profit under equilibrium; second, the copyright duration of 61 years and size of 443 first–copy products are optimal for each creator, given the copyright fee set by copyright authorities in the two countries. If a creator deviates from the duration and size, the creator will incur a loss, given other creators' optimal choices under the copyright fees.

Figure 1. Optimality of Copyright Duration and Creator Size

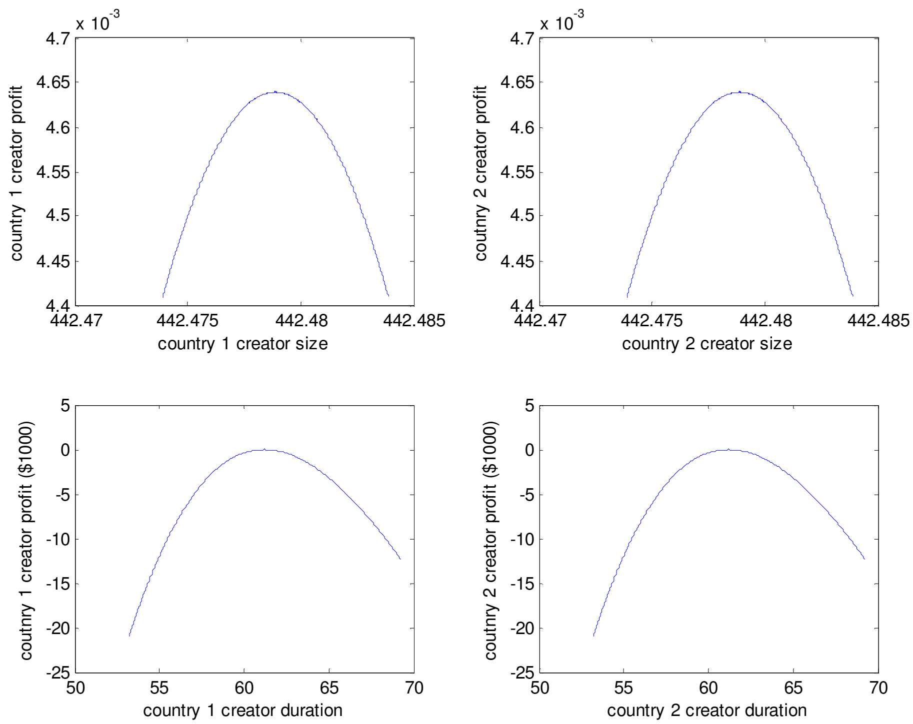
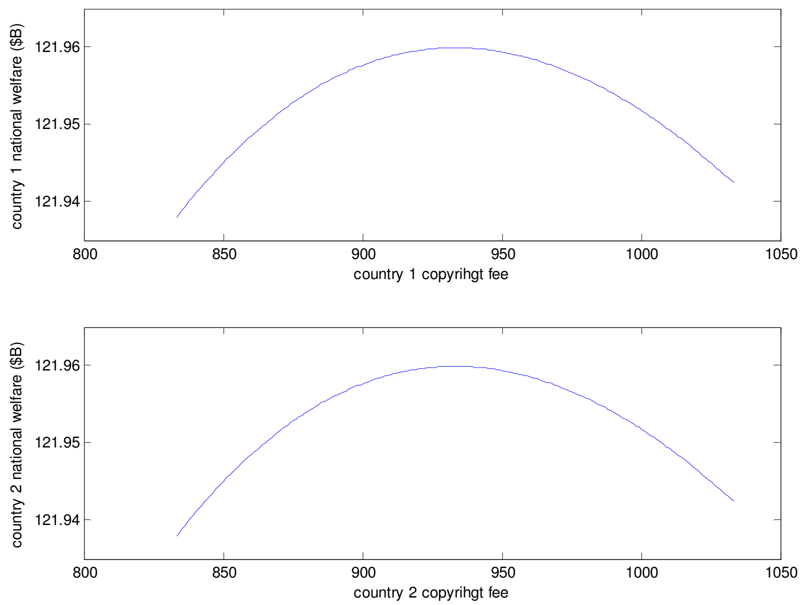


Figure 2: Optimality of Copyright Fee



As shown in Figure 2, the copyright fee of $933 per product per year is optimal for each country, given that the other country sets an optimal fee and creators respond to copyright fees as described by the model. If either country deviates from the optimal copyright fee, the social welfare of the country will be lower than the maximum of $122 billion, given that the other country maintains its optimal fee and the creators behave as described by the model.

Figure 3 further shows that the copyright fees represent the equilibrium of the game of copyright fees between the two countries. The fees are is the intersection of the two reaction functions of each country’s copyright fee reacts to the copyright fee of the other country. The dotted line is the reaction function of copyright fee of country 2 to the copyright fee of country 1. The solid line is that of country 1 to that of country 2. The intersection is the equilibrium copyright fee.

Figure 3: Copyright Fee Equilibrium between the Two Countries

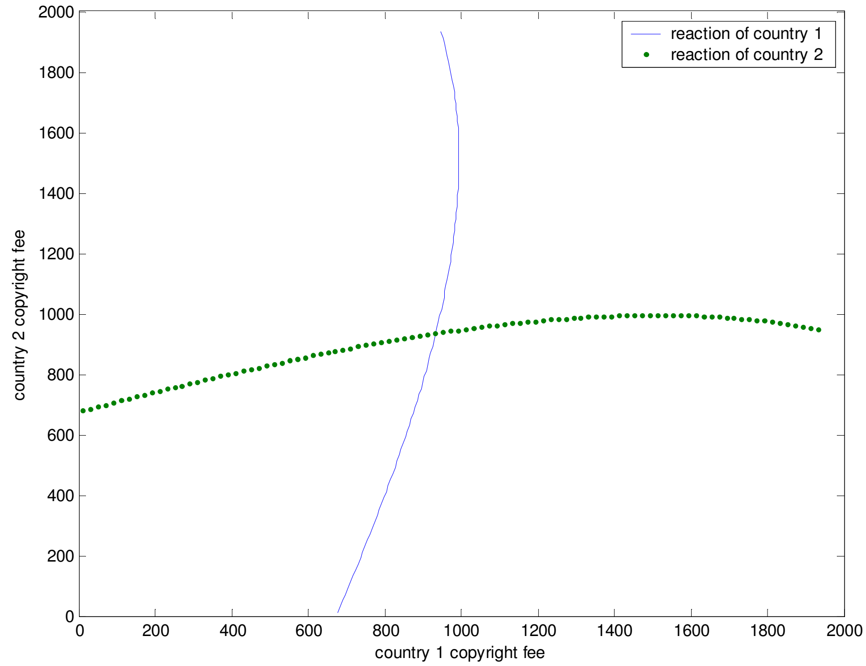


Figure 4. Effect of Copyright Fee of Country 1 on Creator Behavior

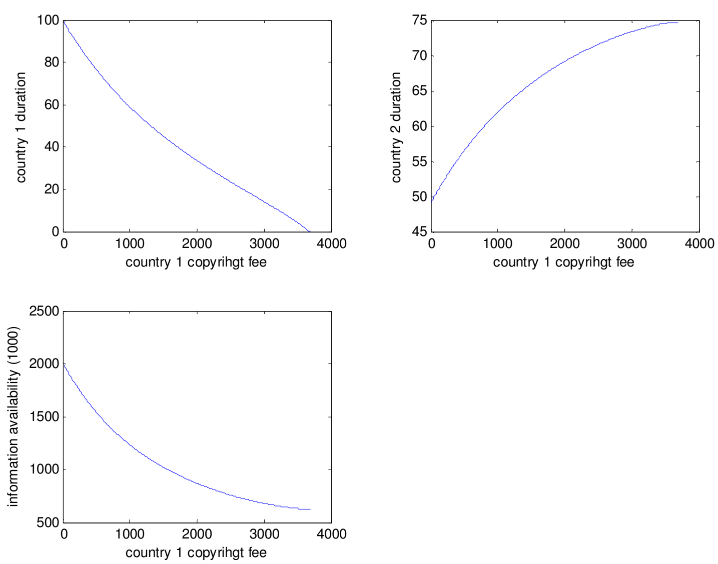


Figure 5. Effect of Copyright Fee of Country 1 on Revenue and Welfare

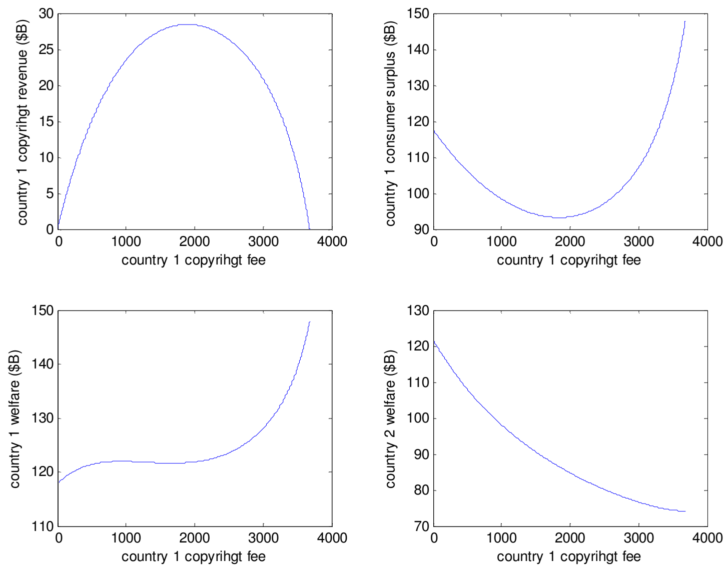


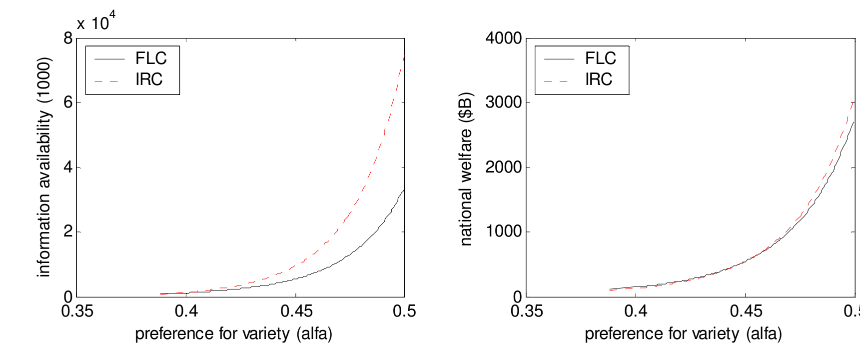
Figure 4 and 5 show the effects of the copyright fee of country 1, given that copyright fee of country 2 stays at the equilibrium level of $933. If country 1 increases its copyright fee, creators will choose shorter copyright duration in the country and choose longer duration in country 2, as renewal of copyright in country 2 becomes relatively cheaper. Creators in the two countries together create fewer first-copy products. Copyright revenue of country 1 first increases with its copyright fee, indicating the effect of higher fee is dominant; it then decreases, indicating the effect of fewer renewals and fewer first-copy products become dominant. The consumer surplus of country 1 first decreases with copyright fee, when the effect of fewer first-copyright products dominates; it then increases, when the effect of reduced loss of consumption due to shorter copyright protection in the country dominates. The national welfare of country 1 reaches a local optimum at copyright fee of $933. National welfare suffers when the country deviates from this fee. However, the optimum is not global. A much larger copyright fee would give the country even higher welfare than the optimum at $933. However, the higher fee is not stable because it is not an equilibrium fee.

In summary, IRC is not necessarily better than FLC in term of social welfare. It can lead to lower welfare in both countries. This is similar to that of Yuan (2009). Second, IRC seems to lead to stronger copyright protection, i.e., longer copyright duration. However, the stronger protection is not necessarily good in terms of social welfare. In the case of the above parameter values, longer protection results in over-supply of original information products and under-consumption of information products.

An important question is whether and how the comparison changes with consumer preference and creative technologies. To answer this question, we change the individual parameter values, while keeping other parameters at the baseline values, and resolve models repeatedly. Solutions are computed for changing the individual parameters for the following ranges: D_1, D_2: $6.724*10^7$-$7.279*10^7$; α: 0.388-0.4995; δ: 1.504-2.204; β: 0.1-0.9; b: 1-50; T_{01}, T_{02}: 92.4-100; θ_1, θ_2: 0.0001-0.3; γ: 0.031038-0.3;, c_{01}, c_{02},: $5.4*10^5$-$3*10^7$; a_1, a_2,: 6370-15940; and ρ_1, ρ_2: 1.1604-1.249104. Solutions for parameter value out these ranges are found difficult to converge. For the all above parameter value changes, copyright duration under IRC remains longer under IRC than under FLC.

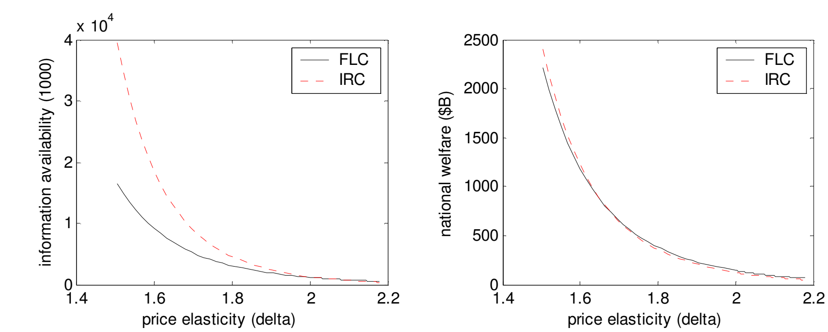
However, the comparison of welfare between IRC and FLC can flip with changes in consumers' preference for variety and the price elasticity of demand for information products. Figure 6 and 7 show the effect of the parameter α and δ on choices of national copyright authorities, those of creators, and market outcome. The figures apply to both country 1 and country 2, as they are based on parameter values which are symmetric for the two countries.

Figure 6. Effect of Consumer Preference for Variety



The parameter α represents preference of consumers for variety of information products. Figure 6 shows that copyright fees set by copyright authority decreases with α; copyright durations under both IRC and FLC increases with α; but that under IRC increases faster than that under FLC, as creators respond both directly to consumer preference and indirectly to lower copyright fees under IRC. The number of first-copy products increases with α both under IRC and FLC; however, it increases faster under IRC for the same reason. National welfare increases with α both under IRC and FLC; but that under IRC increases faster. As a result, when α is smaller than 0.45, national welfare under LFC is higher than under IRC; when α is or is over 0.45, national welfare under IRC is higher than that under FLC.

Figure 7. Effect of Price Elasticity



The parameter δ is the price elasticity of demand. Smaller δ represents stronger demand for information products. Copyright fee decreases when δ decreases or when demand becomes less elastic. Copyright duration increases under both IRC and LFC, when δ decreases; but that under IRC increases faster. The number of first-copy products under both IRC and FLC increases when δ decreases; however, that under IRC increases faster. National welfare increases under both IRC and LFC, when δ decreases; that under IRC increases faster. For δ at or above 1.68, national welfare is higher under FLC. For δ at or below 1.66, national welfare is higher under IRC.

In summary, copyright duration under IRC is longer than the duration under FLC. Furthermore, the longer protection under IRC leads to higher national and global welfare when consumer preference for variety is stronger and demand is inelastic. Otherwise, it leads to lower national and global welfare.

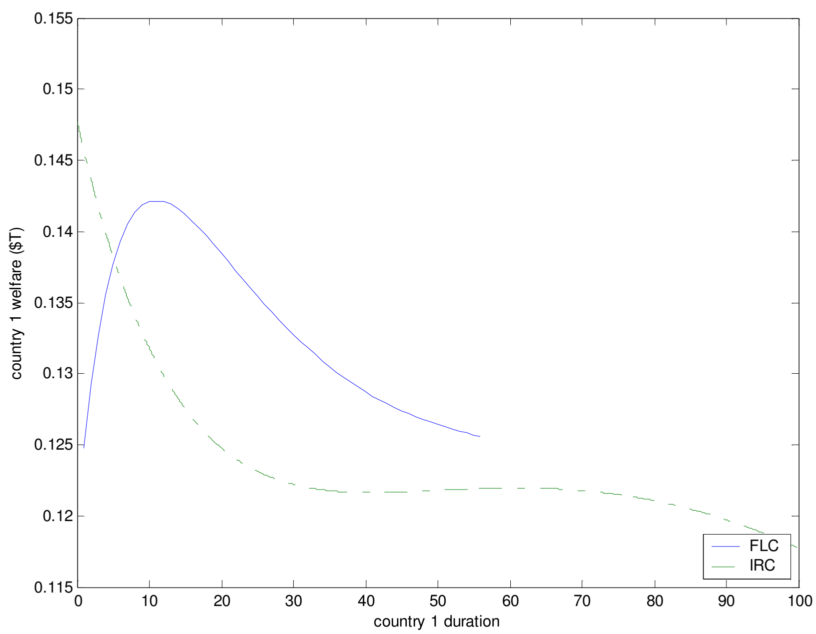
Why may IRC lead to longer protection of copyright than FLC? There may be two reasons. First, the two-country FLC may have a bias toward short copyright protection. Copyright authorities set copyright duration competitively under FLC in two country model. The copyright authority in each country has an incentive to set a short protection in its country to let its consumers to most enjoy the information products and let the other country to provide the protection necessary for creators to create the first-copy products. This may be a critical difference of the two country model from the single country FLC model in (Yuan, 2006).

On the other hand, IRC may have bias toward long copyright protection. IRC lets creators choose the duration of copyright for their products. Creators prefer longer protection. The authorities use copyright fees to induce them to choose the proper length of protection. However, creators may also directly respond to copyright fee in ways other than copyright duration, such as in creative and entry decisions. And they may pass the fees to consumers. When copyright authorities use copyright fee to induce creators to choose a duration they may think as proper, it must consider these “side effects”. These two factors may cause the copyright duration under IRC to be longer than that under FLC.

The result suggests that when preference for variety is weak or demand is elastic, the bias toward long copyright duration of IRC is excessive; IRC leads to lower welfare. On the other side, when preference for variety is strong or demand is inelastic, the bias toward short copyright duration of FLC becomes excessive. FLC leads to lower welfare than IRC.

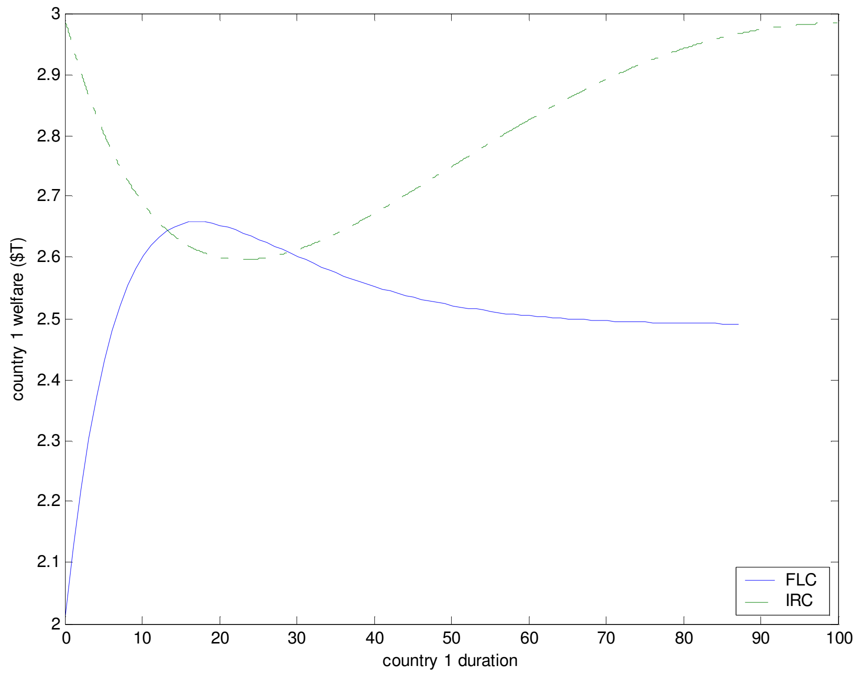
Figure 8 and 9 show the different comparisons of welfare of country 1 between FLC and IRC for two different values of α. Figure 8 is for α=0.4. Figure 9 is for larger α=0.49. In both figures, the welfare of country 1 under LFC is the welfare given that the country 2 stays at the equilibrium duration under FLC for the given values of α: 11 year for α=0.40 and 17 years for α=0.49. The duration for IRC is the duration chosen by creators at various copyright fee in country 1, given that country 2 stays at the equilibrium copyright fee under IRC for the given values of α: $933 for α=0.40 and $4 for α=0.49.

Figure 8. Welfare Comparison of FLC and IRC at Different Copyright Duration (α=0.40)



In figure 8, where α is at smaller 0.40, the optimal copyright duration under FLC is 11 years; the duration under IRC is 61 years, which is induced by a copyright fee of $933 per product per year. And the optimal welfare of country 1 under FLC, reached at duration of 11 years, is $142 billion, higher than that of $122 billion under IRC, reached at the duration of 61 years.

Figure 9. Welfare Comparison of FLC and IRC at Different Copyright Duration (α=0.49)



In figure 9, where α is at bigger 0.49, the optimal copyright duration for country 1 under FLC is 17 years; the optimal copyright duration under IRC is 100 years, induced by a copyright fee of mere $4. And the optimal welfare of country 1 under FLC, reached at duration of 17 years, is $2.7 billion, lower than that of $3.0 trillion under IRC, which is reached at the duration of 100 years.

Conclusion

We compared the welfare of indefinitely renewable copyright (IRC) and the current fixed length copyright (FLC) system in international setting. We did so by developing a two-country model of IRC. In the model, copyright authorities of two countries play a simultaneous game in copyright fee and creators choose copyright duration and make pricing and creative decisions to maximize profit in selling on the global market of information products. The model is compared to a similar model of FLC where copyright authorities play a game in copyright duration and creators make only pricing and creative decisions. The models indicate that national and global welfare is not necessarily higher under IRC than under FLC or vice versa. National and global welfare under IRC can be larger than under FLC when consumer preference for variety is strong or demand is inelastic. Otherwise, welfare under IRC is smaller. Copyright duration under IRC seems to be longer than that under FLC.

The dynamics of copyright policy making in a two-country setting is much more complex than that in the single country setting. Here, we modeled a simultaneous and symmetric game in copyright fees between copyright authorities. They may also play sequential and asymmetric or cooperative games. These other possibilities and more comprehensive comparison of IRC and FLC, which may include transaction cost and other costs of operating the copyright system etc., are left for future studies.

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Appendix: Mathematical Procedures Leading to Numerical Solution

From pricing decision, $\frac{\partial \pi_{1i}}{\partial p_{11it}} = 0, \frac{\partial \pi_{1i}}{\partial p_{12it}} = 0$, $\frac{\partial \pi_{2i}}{\partial p_{21it}} = 0$, and $\frac{\partial \pi_{2i}}{\partial p_{22it}} = 0$, and the demand functions, one easily has:

$$p_{11it} = p_{12it} = p_{21j} = p_{22j} = p \equiv \frac{\delta}{\delta - 1} b \tag{A1}$$

Plug (A1) into demand functions, one has

$$d_{11it} = D_1 s_{1i} \left(\sum_{j=1}^{n_1} s_{1j} + \sum_{i=1}^{n_2} s_{2j} \right)^{\alpha - 1} p^{\beta - \delta} g_1(t) \tag{A2}$$

$$d_{12it} = D_2 s_{1i} \left(\sum_{j=1}^{n_1} s_{1j} + \sum_{i=1}^{n_2} s_{2j} \right)^{\alpha - 1} p^{\beta - \delta} g_2(t) \tag{A3}$$

$$d_{21it} = D_1 s_{2i} \left(\sum_{j=1}^{n_1} s_{1j} + \sum_{i=1}^{n_2} s_{2j} \right)^{\alpha - 1} p^{\beta - \delta} g_1(t) \tag{A4}$$

$$d_{22it} = D_2 s_{2i} \left(\sum_{j=1}^{n_1} s_{1j} + \sum_{i=1}^{n_2} s_{2j} \right)^{\alpha - 1} p^{\beta - \delta} g_2(t) \tag{A5}$$

Plug demand functions (A2-5) into profit functions (1) and (3), the marginal entry condition becomes:

$$\pi_{1i} = [D_1 G_1(T_{11i}) + D_2 G_2(T_{12i})] s_{1i} \left(\sum_{j=1}^{n_1} s_{1j} + \sum_{i=1}^{n_2} s_{2j} \right)^{\alpha - 1} p^{\beta - \delta}(p - b)$$

$$- \frac{s_{1i}}{\gamma} [f_1 \times (1 - e^{-\gamma T_{11i}}) + f_2 \times (1 - e^{-\gamma T_{12i}})] - c_1(s_{1i}) = 0 \tag{A6}$$

$$\pi_{2i} = [D_1 G_1(T_{21i}) + D_2 G_2(T_{22i})] s_{2i} \left(\sum_{j=1}^{n_1} s_{1j} + \sum_{i=1}^{n_2} s_{2j} \right)^{\alpha - 1} p^{\beta - \delta}(p - b)$$

$$-\frac{s_{2i}}{\gamma}[f_1 \times (1 - e^{-\gamma T_{21i}}) + f_2 \times (1 - e^{-\gamma T_{22i}})] - c_2(s_{2i}) = 0 \tag{A7}$$

Where

$$G_k(T) \equiv \int_0^T g_k(t)\, e^{-\gamma t} dt \qquad \text{for k=1, 2} \tag{A8}$$

From the size decisions, $\frac{\partial \pi_{1i}}{\partial s_{1i}} = 0$ and $\frac{\partial \pi_{2i}}{\partial s_{2i}} = 0$, and the marginal profit conditions, $\pi_{1i} = 0$ and $\pi_{2i} = 0$, one has:

$$\left[\frac{1}{s_{1i}} + \frac{(\alpha - 1)}{\sum_{j=1}^{n_1} s_{1j} + \sum_{i=1}^{n_2} s_{2j}}\right]\left[c_1(s_{1i}) + \frac{s_{1i}}{\gamma}[f_1 \times (1 - e^{-\gamma T_{11i}}) + f_2 \times (1 - e^{-\gamma T_{12i}})]\right] =$$

$$c_1{}'(s_{1i}) + \frac{1}{\gamma}[f_1 \times (1 - e^{-\gamma T_{11i}}) + f_2 \times (1 - e^{-\gamma T_{12i}})] \tag{A9}$$

$$\left[\frac{1}{s_{2i}} + \frac{(\alpha - 1)}{\sum_{j=1}^{n_1} s_{1j} + \sum_{i=1}^{n_2} s_{2j}}\right]\left[c_2(s_{2i}) + \frac{s_{2i}}{\gamma}[f_1 \times (1 - e^{-\gamma T_{21i}}) + f_2 \times (1 - e^{-\gamma T_{22i}})]\right] =$$

$$c_2{}'(s_{2i}) + \frac{1}{\gamma}[f_1 \times (1 - e^{-\gamma T_{21i}}) + f_2 \times (1 - e^{-\gamma T_{22i}})] \tag{A10}$$

From identical cost functions (23) and (24) and by symmetry in (A9) and (A10), one has $s_{1i} = s_{1j} \equiv s_1$ and $s_{2i} = s_{2j} \equiv s_2$. Therefore, (A9) and (A10) can be written as:

$$\left[\frac{1}{s_1} + \frac{(\alpha - 1)}{n_1 s_1 + n_2 s_2}\right]\left[c_1(s_1) + \frac{s_1}{\gamma}[f_1 \times (1 - e^{-\gamma T_{11i}}) + f_2 \times (1 - e^{-\gamma T_{12i}})]\right] =$$

$$c_1{}'(s_1) + \frac{1}{\gamma}[f_1 \times (1 - e^{-\gamma T_{11i}}) + f_2 \times (1 - e^{-\gamma T_{12i}})] \tag{A11}$$

$$\left[\frac{1}{s_2}+\frac{(\alpha-1)}{n_1 s_1+n_2 s_2}\right]\left[c_2(s_2)+\frac{s_2}{\gamma}[f_1\times(1-e^{-\gamma T_{21i}})+f_2\times(1-e^{-\gamma T_{22i}})]\right]=$$

$$c_2{}'(s_2)+\frac{1}{\gamma}[f_1\times(1-e^{-\gamma T_{21i}})+f_2\times(1-e^{-\gamma T_{22i}})] \quad \text{(A12)}$$

Profit functions (A6) and (A7) become:

$$\pi_{1i}=[D_1G_1(T_{1i})+D_2G_2(T_{12i})]s_1(n_1s_1+n_2s_2)^{\alpha-1}p^{\beta-\delta}(p-b)$$

$$-\frac{s_1}{\gamma}[f_1\times(1-e^{-\gamma T_{11i}})+f_2\times(1-e^{-\gamma T_{12i}})]-c_1(s_1)=0 \quad \text{(A13)}$$

$$\pi_{2i}=[D_1G_1(T_{21i})+D_2G_2(T_{22i})]s_2(n_1s_1+n_2s_2)^{\alpha-1}p^{\beta-\delta}(p-b)$$

$$-\frac{s_2}{\gamma}[f_1\times(1-e^{-\gamma T_{21i}})+f_2\times(1-e^{-\gamma T_{22i}})]-c_2(s_2)=0 \quad \text{(A14)}$$

From the copyright renewal decisions of creators, $\frac{\partial\pi_{1i}}{\partial T_{11i}}=0$, $\frac{\partial\pi_{1i}}{\partial T_{12i}}=0$, $\frac{\partial\pi_{2i}}{\partial T_{21i}}=0$, and $\frac{\partial\pi_{2i}}{\partial T_{22i}}=0$, one has:

$$D_1g_1(T_{11i})(n_1s_1+n_2s_2)^{\alpha-1}p^{\beta-\delta}(p-b)-f_1=0 \quad \text{(A15)}$$

$$D_2g_2(T_{12i})(n_1s_1+n_2s_2)^{\alpha-1}p^{\beta-\delta}(p-b)-f_2=0 \quad \text{(A16)}$$

$$D_1g_1(T_{21i})(n_1s_1+n_2s_2)^{\alpha-1}p^{\beta-\delta}(p-b)-f_1=0 \quad \text{(A17)}$$

$$D_2g_2(T_{22i})(n_1s_1+n_2s_2)^{\alpha-1}p^{\beta-\delta}(p-b)-f_2=0 \quad \text{(A18)}$$

By symmetry, one has $T_{11i}=T_{11j}=T_{21i}=T_{21j}\equiv T_1$, and $T_{12i}=T_{22j}=T_{22i}=T_{22j}\equiv T_2$. And then creators' renewal decisions become:

$$D_1g_1(T_1)(n_1s_1+n_2s_2)^{\alpha-1}p^{\beta-\delta}(p-b)-f_1=0 \quad \text{(A19)}$$

$$D_2 g_2(T_2)(n_1 s_1 + n_2 s_2)^{\alpha-1} p^{\beta-\delta}(p-b) - f_2 = 0 \tag{A20}$$

Then,

$$S_1 \equiv n_1 s_1 + n_2 s_2 = \left[D_1 p^{\beta-\delta}(p-b)\right]^{-\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{\frac{1}{\alpha-1}} \tag{A21}$$

$$S_2 \equiv n_1 s_1 + n_2 s_2 = \left[D_2 p^{\beta-\delta}(p-b)\right]^{-\frac{1}{\alpha-1}} \left[\frac{f_2}{g_2(T_2)}\right]^{\frac{1}{\alpha-1}} \tag{A22}$$

Demand functions (A2-5) become:

$$d_{11it} = D_1 s_1 (n_1 s_1 + n_2 s_2)^{\alpha-1} p_{11i}{}^{-\delta} p^{\beta} g_1(t) \tag{A23}$$

$$d_{12it} = D_2 s_1 (n_1 s_1 + n_2 s_2)^{\alpha-1} p_{12i}{}^{-\delta} p^{\beta} g_2(t) \tag{A24}$$

$$d_{21it} = D_1 s_2 (n_1 s_1 + n_2 s_2)^{\alpha-1} p_{21i}{}^{-\delta} p^{\beta} g_1(t) \tag{A25}$$

$$d_{22it} = D_2 s_2 (n_1 s_1 + n_2 s_2)^{\alpha-1} p_{22i}{}^{-\delta} p^{\beta} g_2(t) \tag{A26}$$

National welfare (13) and (14) become:

$$W_1 = D_1 (n_1 s_1 + n_2 s_2)^{\alpha} \frac{b^{-\delta+1}}{\delta - 1} p^{\beta} G_1(\infty) - D_1 (n_1 s_1 + n_2 s_2)^{\alpha} \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta - 1} p^{\beta} G_1(T_1)$$

$$+ \frac{1}{\gamma}(n_1 s_1 + n_2 s_2) f_1 (1 - e^{-\gamma T_1}) \tag{A27}$$

$$W_2 = D_2 (n_1 s_1 + n_2 s_2)^{\alpha} \frac{b^{-\delta+1}}{\delta - 1} p^{\beta} G_2(\infty) - D_2 (n_1 s_1 + n_2 s_2)^{\alpha} \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta - 1} p^{\beta} G_2(T_2)$$

$$+ \frac{1}{\gamma}(n_1 s_1 + n_2 s_2) f_2 (1 - e^{-\gamma T_2}) \tag{A28}$$

Marginal entry conditions (A13) and (A14) become:

$$[D_1G_1(T_1) + D_2G_2(T_2)]s_1[S_1]^{\alpha-1}p^{\beta-\delta}(p-b)$$

$$-\tfrac{s_1}{\gamma}[f_1 \times (1 - e^{-\gamma T_1}) + f_2 \times (1 - e^{-\gamma T_2})] - c_1(s_1) = 0 \tag{A29}$$

$$[D_1G_1(T_1) + D_2G_2(T_2)]s_2[S_2]^{\alpha-1}p^{\beta-\delta}(p-b)$$

$$-\tfrac{s_2}{\gamma}[f_1 \times (1 - e^{-\gamma T_{21i}}) + f_2 \times (1 - e^{-\gamma T_{22i}})] - c_2(s_2) = 0 \tag{A30}$$

By (A21) and (A22), national welfare functions (A27) and (A28) can be rewritten as:

$$W_1 = D_1(S_1)^\alpha \frac{b^{-\delta+1}}{\delta-1}p^\beta G_1(\infty) - D_1(S_1)^\alpha \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta-1}p^\beta G_1(T_1)$$

$$+\tfrac{1}{\gamma}(S_1)f_1(1 - e^{-\gamma T_1}) \tag{A31}$$

$$W_2 = D_2(S_2)^\alpha \frac{b^{-\delta+1}}{\delta-1}p^\beta G_2(\infty) - D_2(S_2)^\alpha \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta-1}p^\beta G_2(T_2)$$

$$+\tfrac{1}{\gamma}(S_2)f_2(1 - e^{-\gamma T_2}) \tag{A32}$$

(A11), (A12), (A21), (A22), and (A29-A30) make s_1, s_2, T_1, and T_2 functions of f_1 and f_2. Therefore, W_1 and W_2 are only functions of f_1 and f_2.

First-order condition of maximizing W_1 with respect to f_1, given f_2 is:

$$\frac{\partial W_1}{\partial f_1} = D_1 \frac{b^{-\delta+1}}{\delta-1}p^\beta G_1(\infty)\alpha {S_1}^{\alpha-1}\left[\frac{\partial S_1}{\partial f_1}\right] - D_1 \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta-1}p^\beta \alpha {S_1}^{\alpha-1}\frac{\partial S_1}{\partial f_1}G_1(T_1)$$

$$-D_1 \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta-1}p^\beta {S_1}^\alpha g_1(T_1)e^{-\gamma T_1}\frac{\partial T_1}{\partial f_1}$$

$$+\tfrac{1}{\gamma}\left(\tfrac{\partial S_1}{\partial f_1}\right)f_1(1 - e^{-\gamma T_1}) + \tfrac{1}{\gamma}(S_1)(1 - e^{-\gamma T_1}) + (S_1)f_1 e^{-\gamma T_1}\tfrac{\partial T_1}{\partial f_1} = 0 \tag{A33}$$

First-order condition of maximizing W_2 with respect to f_2, given f_1 is:

$$\frac{\partial W_2}{\partial f_2} = D_2 \frac{b^{-\delta+1}}{\delta - 1} p^{\beta} G_2(\infty) \alpha {S_2}^{\alpha-1} \left[\frac{\partial S_2}{\partial f_2}\right] - D_2 \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta - 1} p^{\beta} \alpha {S_2}^{\alpha-1} \frac{\partial S_2}{\partial f_2} G_2(T_2)$$

$$-D_2 \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta - 1} p^{\beta} {S_2}^{\alpha} g_2(T_2) e^{-\gamma T_2} \frac{\partial T_2}{\partial f_2}$$

$$+\frac{1}{\gamma}\left(\frac{\partial S_2}{\partial f_2}\right) f_2 (1 - e^{-\gamma T_2}) + \frac{1}{\gamma}(S_2)(1 - e^{-\gamma T_2}) + (S_2) f_2 e^{-\gamma T_2} \frac{\partial T_2}{\partial f_2} = 0 \quad \text{(A34)}$$

We need to get the derivatives of s_1, s_2, T_1, T_2, S_1 and S_2 to f_1 and f_2 by the implicit function theorem from(A11-A12),(A21-A22), and (A29-A30).

By (A21) and (A22),

$$\frac{\partial S_1}{\partial f_1} = \frac{1}{\alpha-1}\left[D_1 p^{\beta-\delta}(p-b)\right]^{-\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{\frac{2-\alpha}{\alpha-1}} \left[\frac{1}{g_1(T_1)} - \frac{f_1}{{g_1}^2(T_1)} {g_1}'(T_1) \frac{\partial T_1}{\partial f_1}\right] \quad \text{(A35)}$$

$$\frac{\partial S_2}{\partial f_2} = \frac{1}{\alpha-1}\left[D_2 p^{\beta-\delta}(p-b)\right]^{-\frac{1}{\alpha-1}} \left[\frac{f_2}{g_2(T_2)}\right]^{\frac{2-\alpha}{\alpha-1}} \left[\frac{1}{g_2(T_2)} - \frac{f_2}{{g_2}^2(T_2)} {g_2}'(T_2) \frac{\partial T_2}{\partial f_2}\right] \quad \text{(A36)}$$

Collect the constraints (A11-A12) and (A29-A30):

$$H_1 \equiv \left\{\frac{1}{s_1} + (\alpha-1)\left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{1}{\alpha-1}}\right\} c_1(s_1)$$

$$+\frac{\left\{(\alpha-1)\left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{1}{\alpha-1}}\right\} s_1}{\gamma} \left[f_1 \times (1 - e^{-\gamma T_1}) + f_2 \times (1 - e^{-\gamma T_2})\right]$$

$$-{c_1}'(s_1) = 0 \quad \text{(A37)}$$

$$H_2 \equiv \left\{\frac{1}{s_2} + (\alpha-1)\left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}}\left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{1}{\alpha-1}}\right\} c_2(s_2)$$

$$+\frac{\left\{(\alpha-1)\left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}}\left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{1}{\alpha-1}}\right\} s_2}{\gamma}\left[f_1 \times (1-e^{-\gamma T_1}) + f_2 \times (1-e^{-\gamma T_2})\right]$$

$$-c_2{}'(s_2) = 0 \qquad \text{(A38)}$$

$$H_3 \equiv \frac{1}{D_1}\left[D_1 G_1(T_1) + D_2 G_2(T_2)\right]\frac{f_1}{g_1(T_1)} - \frac{1}{\gamma}\left[f_1 \times (1-e^{-\gamma T_1}) + f_2 \times (1-e^{-\gamma T_2})\right] - \frac{c_1(s_1)}{s_1} = 0$$

(A39)

$$H_4 = \frac{1}{D_2}\left[D_1 G_1(T_1) + D_2 G_2(T_2)\right]\frac{f_2}{g_2(T_2)} - \frac{1}{\gamma}\left[f_1 \times (1-e^{-\gamma T_1}) + f_2 \times (1-e^{-\gamma T_2})\right] - \frac{c_2(s_2)}{s_2} = 0$$

(A40)

$$A_{11} \equiv \frac{\partial H_1}{\partial s_1} = \left\{\frac{1}{s_1} + (\alpha-1)\left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}}\left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{1}{\alpha-1}}\right\} c_1{}'(s_1) - \frac{1}{s_1{}^2} c_1(s_1)$$

$$+\frac{\left\{(\alpha-1)\left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}}\left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{1}{\alpha-1}}\right\}}{\gamma}\left[f_1 \times (1-e^{-\gamma T_1}) + f_2 \times (1-e^{-\gamma T_2})\right]$$

$$-c_1{}''(s_1) \qquad \text{(A41)}$$

$$A_{12} \equiv \frac{\partial H_1}{\partial s_2} = 0 \qquad \text{(A42)}$$

$$A_{13} \equiv \frac{\partial H_1}{\partial T_1} = \left\{ \left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{\alpha}{\alpha-1}} \frac{f_1}{{g_1}^2(T_1)} {g_1}^{'}(T_1) \right\} c_1(s_1)$$

$$+ \frac{\left\{ \left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{\alpha}{\alpha-1}} \frac{f_1}{{g_1}^2(T_1)} {g_1}^{'}(T_1) \right\} s_1}{\gamma} [f_1 \times (1 - e^{-\gamma T_1}) + f_2 \times (1 - e^{-\gamma T_2})]$$

$$+(\alpha-1)\left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{1}{\alpha-1}} s_1[f_1 \times (e^{-\gamma T_1})] \quad \text{(A43)}$$

$$A_{14} \equiv \frac{\partial H_1}{\partial T_2} = +(\alpha-1)\left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{1}{\alpha-1}} s_1[f_2 \times (e^{-\gamma T_2})] \quad \text{(A44)}$$

$$a_1 \equiv \frac{\partial H_1}{\partial f_1} = \left\{ -\left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{\alpha}{\alpha-1}} \frac{1}{g_1(T_1)} \right\} c_1(s_1)$$

$$+ \frac{\left\{ -\left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{\alpha}{\alpha-1}} \frac{1}{g_1(T_1)} \right\} s_1}{\gamma} [f_1 \times (1 - e^{-\gamma T_1}) + f_2 \times (1 - e^{-\gamma T_2})]$$

$$+ \frac{\left\{ (\alpha-1)\left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{1}{\alpha-1}} \right\} s_1}{\gamma} [(1 - e^{-\gamma T_1})] \quad \text{(A45)}$$

$$b_1 \equiv \frac{\partial H_1}{\partial f_2} = \frac{\left\{ (\alpha-1)\left[D_1 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_1}{g_1(T_1)}\right]^{-\frac{1}{\alpha-1}} \right\} s_1}{\gamma} [(1 - e^{-\gamma T_2})] \quad \text{(A46)}$$

$$A_{21} \equiv \frac{\partial H_2}{\partial s_1} = 0 \quad \text{(A47)}$$

$$A_{22} \equiv \frac{\partial H_2}{\partial s_2} = \left\{ \frac{1}{s_2} + (\alpha - 1)\left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{1}{\alpha-1}} \right\} c_2{}'(s_2) - \frac{1}{s_2{}^2} c_2(s_2)$$

$$+ \frac{(\alpha - 1)\left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{1}{\alpha-1}}}{\gamma} [f_1 \times (1 - e^{-\gamma T_1}) + f_2 \times (1 - e^{-\gamma T_2})]$$

$$-c_2{}''(s_2) \qquad \text{(A48)}$$

$$A_{23} \equiv \frac{\partial H_2}{\partial T_1} = +(\alpha - 1)\left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{1}{\alpha-1}} s_2 [f_1 \times (e^{-\gamma T_1})] \qquad \text{(A49)}$$

$$A_{24} \equiv \frac{\partial H_2}{\partial T_2} = \left\{ \left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{\alpha}{\alpha-1}} \frac{f_2}{g_2{}^2(T_2)} g_2{}'(T_2) \right\} c_2(s_2)$$

$$+ \frac{\left\{ \left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{\alpha}{\alpha-1}} \frac{f_2}{g_2{}^2(T_2)} g_2{}'(T_2) \right\} s_2}{\gamma} [f_1 \times (1 - e^{-\gamma T_1}) + f_2$$

$$\times (1 - e^{-\gamma T_2})]$$

$$+(\alpha - 1)\left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{1}{\alpha-1}} s_2 [f_2 \times (e^{-\gamma T_2})] \qquad \text{(A50)}$$

$$a_2 \equiv \frac{\partial H_2}{\partial f_1} = \frac{\left\{ (\alpha-1)\left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{1}{\alpha-1}} \right\} s_2}{\gamma} [(1 - e^{-\gamma T_1})] \qquad \text{(A51)}$$

$$b_2 \equiv \frac{\partial H_2}{\partial f_2} = \left\{ -\left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}} \left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{\alpha}{\alpha-1}} \frac{1}{g_2(T_2)} \right\} c_2(s_2)$$

$$+\frac{\left\{-\left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}}\left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{\alpha}{\alpha-1}}\frac{1}{g_2(T_2)}\right\}s_2}{\gamma}\left[f_1\times(1-e^{-\gamma T_1})+f_2\times(1-e^{-\gamma T_2})\right]$$

$$+\frac{\left\{(\alpha-1)\left[D_2 p^{\beta-\delta}(p-b)\right]^{\frac{1}{\alpha-1}}\left[\frac{f_2}{g_2(T_2)}\right]^{-\frac{1}{\alpha-1}}\right\}s_2}{\gamma}\left[(1-e^{-\gamma T_2})\right] \quad \text{(A52)}$$

$$A_{31}\equiv\frac{\partial H_3}{\partial s_1}=-\frac{c'_1(s_1)}{s_1}+\frac{c_1(s_1)}{{s_1}^2} \quad \text{(A53)}$$

$$A_{32}\equiv\frac{\partial H_3}{\partial s_2}=0 \quad \text{(A54)}$$

$$A_{33}\equiv\frac{\partial H_3}{\partial T_1}=-\frac{1}{D_1}\left[D_1G_1(T_1)+D_2G_2(T_2)\right]\frac{f_1}{g^2{}_1(T_1)}g'_1(T_1) \quad \text{(A55)}$$

$$A_{34}\equiv\frac{\partial H_3}{\partial T_2}=\frac{1}{D_1}\left[D_2g_2(T_2)e^{-\gamma T_2}\right]\frac{f_1}{g_1(T_1)}-\left[f_2\times(e^{-\gamma T_2})\right] \quad \text{(A56)}$$

$$a_3\equiv\frac{\partial H_3}{\partial f_1}=\frac{1}{D_1}\left[D_1G_1(T_1)+D_2G_2(T_2)\right]\frac{1}{g_1(T_1)}-\frac{1}{\gamma}\left[(1-e^{-\gamma T_1})\right] \quad \text{(A57)}$$

$$b_3\equiv\frac{\partial H_3}{\partial f_2}=-\frac{1}{\gamma}\left[(1-e^{-\gamma T_2})\right] \quad \text{(A58)}$$

$$A_{41}\equiv\frac{\partial H_4}{\partial s_1}=0 \quad \text{(A59)}$$

$$A_{42}\equiv\frac{\partial H_4}{\partial s_2}=-\frac{c'_2(s_2)}{s_2}+\frac{c_2(s_2)}{{s_2}^2} \quad \text{(A60)}$$

$$A_{43}\equiv\frac{\partial H_4}{\partial T_1}=\frac{1}{D_2}\left[D_1g_1(T_1)e^{-\gamma T_1}\right]\frac{f_2}{g_2(T_2)}-\left[f_1\times(e^{-\gamma T_1})\right] \quad \text{(A61)}$$

$$A_{44}\equiv\frac{\partial H_4}{\partial T_2}=-\frac{1}{D_2}\left[D_1G_1(T_1)+D_2G_2(T_2)\right]\frac{f_2}{g^2{}_2(T_2)}g'_2(T_2) \quad \text{(A62)}$$

$$a_4 \equiv \frac{\partial H_4}{\partial f_1} = -\frac{1}{\gamma}[(1 - e^{-\gamma T_1})] \tag{A63}$$

$$b_4 \equiv \frac{\partial H_4}{\partial f_2} = \frac{1}{D_2}[D_1 G_1(T_1) + D_2 G_2(T_2)]\frac{1}{g_2(T_2)} - \frac{1}{\gamma}[(1 - e^{-\gamma T_2})] \tag{A64}$$

By implicit function theorem,

$$\begin{bmatrix} \frac{\partial s_1}{\partial f_1} \\ \frac{\partial s_2}{\partial f_1} \\ \frac{\partial T_1}{\partial f_1} \\ \frac{\partial T_2}{\partial f_1} \end{bmatrix} = -\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \tag{A65}$$

and

$$\begin{bmatrix} \frac{\partial s_1}{\partial f_2} \\ \frac{\partial s_2}{\partial f_2} \\ \frac{\partial T_1}{\partial f_2} \\ \frac{\partial T_2}{\partial f_2} \end{bmatrix} = -\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \tag{A66}$$

(A65-A66) are only functions of s_1, s_2, T_1, T_2, f_1, f_2. So (A35-A36) becomes only functions of s_1, s_2, T_1, T_2, f_1, f_2. Therefore, (A33-A34) becomes functions of only of s_1, s_2, T_1, T_2, f_1, f_2:

$$H_5 \equiv \frac{\partial W_1}{\partial f_1} = D_1 \frac{b^{-\delta+1}}{\delta - 1} p^\beta G_1(\infty)\alpha {S_1}^{\alpha-1}\left[\frac{\partial S_1}{\partial f_1}\right] - D_1 \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta - 1} p^\beta \alpha {S_1}^{\alpha-1}\frac{\partial S_1}{\partial f_1} G_1(T_1)$$

$$-D_1 \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta - 1} p^\beta {S_1}^\alpha g_1(T_1) e^{-\gamma T_1}\frac{\partial T_1}{\partial f_1}$$

$$+\frac{1}{\gamma}\left(\frac{\partial S_1}{\partial f_1}\right) f_1(1 - e^{-\gamma T_1}) + \frac{1}{\gamma}(S_1)(1 - e^{-\gamma T_1}) + (S_1) f_1 e^{-\gamma T_1}\frac{\partial T_1}{\partial f_1} = 0 \tag{A67}$$

$$H_6 \equiv \frac{\partial W_2}{\partial f_2} = D_2 \frac{b^{-\delta+1}}{\delta - 1} p^\beta G_2(\infty)\alpha {S_2}^{\alpha-1}\left[\frac{\partial S_2}{\partial f_2}\right] - D_2 \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta - 1} p^\beta \alpha {S_2}^{\alpha-1}\frac{\partial S_2}{\partial f_2} G_2(T_2)$$

$$-D_2 \frac{b^{-\delta+1} - p^{-\delta+1}}{\delta - 1} p^{\beta} {S_2}^{\alpha} g_2(T_2) e^{-\gamma T_2} \frac{\partial T_2}{\partial f_2}$$

$$+\frac{1}{\gamma}\left(\frac{\partial S_2}{\partial f_2}\right) f_2 (1 - e^{-\gamma T_2}) + \frac{1}{\gamma}(S_2)(1 - e^{-\gamma T_2}) + (S_2) f_2 e^{-\gamma T_2} \frac{\partial T_2}{\partial f_2} = 0 \qquad \text{(A68)}$$

(A37-A40) and (A67-A68) are six equations for six variables of s_1, s_2, T_1, T_2, f_1, f_2. They can be solved by Newton's Method:

$$\begin{bmatrix} s_1 \\ s_2 \\ T_1 \\ T_2 \\ f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} s_{10} \\ s_{20} \\ T_{10} \\ T_{20} \\ f_{10} \\ f_{20} \end{bmatrix} - J^{-1} \times \begin{bmatrix} H_{10} \\ H_{20} \\ H_{30} \\ H_{40} \\ H_{50} \\ H_{60} \end{bmatrix} \qquad \text{(A69)}$$

Where J is the Jacobian matrix of H_1, H_2, H_3, H_4, H_5, H_6.