Piracy and Quality Choice in Monopolistic Markets*

Matteo Alvisi
Department of Economics, University of Bologna

Elena Argentesi
European University Institute, Florence

Emanuela Carbonara
Department of Economics, University of Bologna

April 2002

Abstract

We study the impact of piracy on the quality choices of a monopolist. In the absence of piracy, the monopolist has no incentive to differentiate its products. With piracy the monopolist might instead produce more than one quality, so that differentiation arises as the optimal strategy. This is because the producer wants to divert consumers from the pirated good to the original one. Differentiation involves either producing a low-quality good such that piracy is eliminated or the monopolist can choose qualities such that piracy is still observed in equilibrium.

JEL Classification Numbers: L12, L15, L82, L86.

Keywords: Product differentiation, Multiproduct monopolist, Quality, Piracy.

*We are grateful to Massimo Motta for comments, discussions and advice, as well as to Giacomo Calzolari, Vincenzo Denicolò, Paolo Garella and Luca Lambertini for comments and suggestions.

1 Introduction

The huge increase of piracy and private copying is a phenomenon that in recent years has greatly affected the market for information goods, and specifically digital goods such as software and music compact disks. The widespread practice of file-sharing through the Internet, together with the improvements and the greater availability of copying technologies (like peer-to-peer connections), lead to a boom of unauthorized copies.

The reproduction of digital information differs from the reproduction of non-digital media such as journals, books, audio and video cassettes. Digital media are characterized by a "vertical" pattern of reproduction,[1] since it is enough to have an original and then copies can be made out of copies without a progressive decline in their quality. Therefore in markets for digital goods the traditional result of "indirect appropriability" cannot apply.[2] With indirect appropriability the seller can extract the rents from all users by charging a higher price for the original since their total willingness to pay as a whole is higher than the single buyer's. When quality does not decline with the number of copies the price that extracts all surplus from all users would be too high to be affordable by the single buyer. These features enhance the harmful impact of piracy on profits.[3]

From a legal perspective, one way in which firms can counteract this phenomenon is by undertaking legal actions against the infringements of copyright laws (the most recent case is the suit brought by the five major record companies against Napster) and by lobbying for stricter copyright laws.[4] The goal of our paper is instead to examine from an economic perspective the impact of piracy on the business strategies of a firm.[5]

We study how piracy affects the firm's incentives to vertically differentiate the quality of its products. In general the presence of piracy reduces the demand for the original good and therefore profits. The monopolist could then introduce a low-quality good in order to capture at least part of the demand for piracy, shifting it from the illegal market, where copies of the

[1] For a classification and description of information reproduction, see Shy (2001).

[2] This concept was first introduced by Liebowitz (1985).

[3] As pointed out by Shy and Thisse (1999), the negative impact of copying on producers' profits can be mitigated by the presence of network externalities, *i.e.* when consumers' utility increases in the number of other consumers using the good (either bought or copied). This assumption is quite plausible with respect to software but it is hardly applicable to other information goods such as music and printed items.

[4] For instance, the European Union has recently issued a directive on copyright protection of online digital goods (Directive 2001/29/CE).

[5] We consider a situation where piracy cannot be completely eliminated by copyright protection laws.

high-quality good are exchanged.

In practice, producing a new, low-quality good means that the firm introduces a reduced version of what was previously produced without some of its features. Examples of vertical differentiation in the software market are the reduced versions released as shareware or freeware on the Internet. In classical music, lower-quality (and lower-cost) CDs are sold without a booklet and performed by less famous orchestras. Similarly, music companies are developing technologies to sell music on-line[6] (without package and without a booklet with photos, lyrics and information on the artists) as a way to capture consumers with low willingness to pay who could otherwise be potential pirates.

In our model, the goods produced by the monopolist can be obtained in two different ways: consumers can either purchase them in the original version or pay for unauthorized copies. Our definition of a copy includes both a copy made by somebody else and bought illegally (*i.e.* a CD from an unauthorized vendor) and a copy made by the consumer herself (*i.e.* a CD copied with a CD burner). Given that we refer mainly to digital goods, we assume that the original good and its copies are identical in terms of quality.

We also assume that consumers are heterogeneous in two respects. First, there is a continuum of consumers identical in tastes but with different preferences for quality. Second, consumers are characterized by two different costs of pirating (or going to the illegal market), under the assumption that a higher cost of copying is associated with a higher willingness to pay for quality.

Our main results are as follows. In the absence of piracy, the monopolist has no incentive to differentiate its products. With piracy the monopolist might instead produce more than one quality, so that differentiation arises as the optimal strategy.

The analysis involves two distinct cases. If the proportion of consumers with high cost of pirating is low and the monopolist differentiates its products, prices are set such that the high-cost consumers buy the high-quality good, whereas low-cost consumers will either buy the low-quality good or pirate. In this case, vertical differentiation will be more profitable than no differentiation when the heterogeneity of consumers with respect to their cost of pirating is sufficiently high. Here, the monopolist indeed fights pirates through vertical differentiation, producing a new and lower quality that eliminates demand for copies and hence piracy. When instead consumers are more similar in their costs of pirating, the monopolist will fight piracy by set-

[6] The five Majors have recently made two joint ventures to sell music online, namely MusicNet and Pressplay.

ting a price low enough such that everybody buys the good, and no vertical differentiation takes place.

If the proportion of high-cost consumers is high and the monopolist produces two goods, prices are set such that high-cost consumers buy both goods. In this situation, an interesting trade-off in the choice of quality levels arises. The monopolist still has an incentive to offer a low quality level that is high enough to attract consumers with low willingness to pay. However, if this level is high enough, some high-cost consumers may switch from the high- to the low-quality good. This second effect may counterbalance the incentive to eliminate piracy by increasing the low quality level and, in equilibrium, the monopolist might admit some piracy while still differentiating.

Our work builds on the literature on copying and piracy. In particular, it is related to a recent paper by Gayer and Shy (2001) that analyzes the incentives of publishers to distribute via the Internet versions of digital goods that compete with products sold in stores. Under the assumption of network externalities between buyers of the original good and "downloaders" from the Internet, they show that the introduction of a product of (exogenously-given) lower quality over the Internet is profit-enhancing for the monopolist. The issue of copyright protection in the presence of network externalities is also addressed by Conner and Rumelt (1991) and, in an horizontally differentiated duopoly, by Shy and Thisse (1999).

One major difference of our approach relative to the previous literature is that we endogenize the choice of product quality by making use of a model of vertical differentiation. Moreover, we do not assume network externalities between buyers of the original good and "downloaders" from the Internet, which instead drive Gayer and Shy's results.[7] The previous models showed that the distribution of a lower-quality good over the Internet may be profitable for the producer because it may raise the demand for the original good through demand-side externalities. On the contrary, in our framework the introduction of a lower quality allows the producer to divert some consumers from copying the good to buying an original (lower-quality) version and is therefore a way to reduce (or eliminate) piracy.

Our model is more generally related to the literature on copying (e.g., Johnson, 1985; Liebowitz, 1985; Besen and Kirby, 1989; Varian, 2000). These papers generally assume that copies can be made only from originals[8] and therefore producers can appropriate some of the consumer surplus from illegal

[7]Indeed, Hui et al. (2001) tested empirically the impact of piracy on the legitimate demand for recorded music and found that the potential positive effects of piracy (network externalities, sampling, sharing, and others), if they exist, do not compensate for the direct loss of customers.

[8]Alternatively, copies of copies have a lower quality than copies made from originals.

copies.[9] Therefore, even in the absence of network externalities, copying may be profitable because of indirect appropriability. Our paper instead rules out any potential beneficial effect of piracy for producers (either through some kind of appropriability or through network effects) and studies the pricing policies of a monopolist faced with this problem.

Our paper is also an application of the theoretical literature on vertical differentiation by a monopolist. The main idea of this literature is that a monopolist may want to segment the market by offering different qualities of the same good in order to extract more consumer surplus. Two classes of models investigate different assumptions on quality costs.[10] Gabszewicz et al. (1986), building on the model by Shaked and Sutton (1982), assume fixed costs of quality improvement and show that the number of products sold depends on the dispersion of consumers' incomes. Mussa and Rosen (1982), Itoh (1983), Maskin and Riley (1984), and Lambertini (1987) study instead the case where unit production costs are increasing in quality, and find that the monopolist has an incentive to produce a broad range of qualities in order to segment the market.

Our contribution to this literature is that we propose an alternative explanation for vertical differentiation, namely piracy. We do this by focusing on a case where no vertical differentiation would arise in an equilibrium without piracy. In this setting, we show that when piracy is instead possible, it may be more profitable to introduce a lower quality rather than to produce one quality only.

The paper is organized as follows. Section 2 presents the base model. In sections 3 and 4 we analyze the price and quality strategies of a monopolist producing a single product and two products respectively. Section 5 compares the profits of vertical differentiation with those resulting from the strategy of a single quality level and derives conditions for the introduction of a lower quality. Section 6 concludes.

2 The model

We assume that consumers can obtain the good in two different ways: they can purchase its original version in the legal market or, alternatively, obtain a copy in an illegal market, where only unauthorized copies are available.[11]

[9] Therefore these models are applicable mainly to non-digital goods.

[10] For a model that considers both types of assumptions on costs in an oligopolistic setting, see Motta (1993).

[11] This is what we label *piracy* in the rest of the paper, although we disregard copyright issues and related legal punishments.

Examples of goods where illegal copies exist include music CDs, software, videos and also design clothes and accessories.

2.1 Production

We consider a monopolist operating in a market where consumers have the possibility of pirating the good (or obtaining a copy on the illegal market).[12] Piracy reduces demand in the legal market and potentially firms' profits. When choosing its business strategies, the monopolist tries to counter this problem. Throughout the paper, we assume that price discrimination for products of the same quality is not an available option for the monopolist (for instance, because arbitrage is possible). However, the monopolist can choose to vertically differentiate its goods. The monopolist has two main strategies: it can produce one good only, whose quality and price are q and p, or two goods, whose qualities are q_1 and q_2, with $q_1 < q_2$, and $q_i \in [0, +\infty)$, $i = 1, 2$. Let p_1 denote the price of the good of quality q_1 and p_2 the price of quality q_2. We assume that the firm has neither variable production costs nor quality-dependent (fixed) costs.[13]

We are looking at a three-stage decisional process where the monopolist first chooses whether to produce one or two qualities, in the second stage quality levels are set and in the third it chooses market price(s).[14]

2.2 Consumers

Consumers are heterogeneous in two respects. First, we assume a continuum of consumers identical in tastes but with different preferences for quality θ (like in Mussa and Rosen (1978)). Consumers are uniformly distributed over the interval $[\underline{\theta}, \overline{\theta}]$, with $\underline{\theta} > 0$. Second, consumers differ in their cost of pirating the good. This assumption is introduced to ensure the production of

[12] We consider a situation where piracy cannot be completely eliminated by copyright protection laws.

[13] As to the variable costs, it would be equivalent to assume a constant marginal cost of production. The absence of quality-dependent fixed costs, besides making the analysis more tractable, helps us to isolate the impact of piracy on the incentives to differentiate from that generated by a costly production of quality. It is in fact a well established result that a monopolist differentiates its quality in the presence of quality costs (see Spence, 1975 and Mussa and Rosen, 1978).

[14] This assumption on the structure of the decisional process is intended to capture the idea that the price can in practice be varied at will, while a change in the quality specification of the good involves a modification of the appropriate "production facilities" (Shaked and Sutton, 1982).

an high-quality good in equilibrium.[15] A first group of consumers sustains a high cost c_H when buying a non-original copy of the good. Consumers with cost c_H have also high preference for quality $\theta \in [\hat{\theta}, \overline{\theta}]$. A second group of consumers has a low cost of copying, c_L, with $c_H > c_L$. Consumers with cost c_L have preference for quality $\theta \in [\underline{\theta}, \hat{\theta}]$. Assuming that a higher cost of copying is associated with higher taste for quality is rather plausible and empirically relevant.[16] This is even clearer if we reinterpret consumers' heterogeneity not in terms of taste for quality, but rather in terms of income. In fact, c_i can also be considered as the opportunity cost of going to the illegal market or spending time and resources to make a copy. Under this interpretation, richer people (with high willingness to pay for quality) usually prefer to buy the original rather than a pirated good.

We are examining a case where all individuals consume the good, either purchased or copied (we call this "covered market", even though not all consumers necessarily buy the good but some pirate it).[17] Each consumer consumes one unit of the good. The utility of a consumer of type $i = H, L$ with preference for quality $\theta \in [\underline{\theta}, \overline{\theta}]$ when she legally buys quality k is

$$U(\theta, q_k, p_k) = \theta q_k - p_k \tag{1}$$

whereas, when she obtains a copy is

$$U(\theta, q_k, c_i) = \theta q_k - c_i \tag{2}$$

[15] If consumers were identical in their costs of copying, c, and the monopolist produces two qualities at prices $p_1$and p_2, $p_1 < p_2$ the following cases are possible. i) If $c < p_1 < p_2$, consumers would rather pirate both goods than purchase them. The demand for both goods is zero; ii) if $p_1 \leq c < p_2$, no consumer is willing to buy q_2, so that the latter is not produced; iii) if $p_1 < p_2 \leq c$, prices are so low that no consumer will pirate. The monopolist will then produce a single quality level.

It is then clear that any price strategy eliminates both piracy and the incentive for vertical differentiation. Therefore, only when consumers are heterogeneous in their cost c, product differentiation can be profitable.

[16] We also examined a case in which the relationship between the cost of pirating c_i and the preference for quality θ is negative, i.e. consumers with $\theta \in [\underline{\theta}, \hat{\theta}]$ have a high cost of pirating (c_L) while those with $\theta \in [\hat{\theta}, \overline{\theta}]$ sustain a cost c_H. Results are qualitatively similar to those presented in this article and vertical differentiation might still be an equilibrium strategy of the firm.

[17] The assumption of a covered market configuration when the monopolist is producing a single quality is actually not restrictive in the presence of piracy. In fact, it turns out that the uncovered market configuration is less profitable than letting all individuals consume the good. More specifically, when the market is not fully covered, profits are $\Pi = \frac{p}{\overline{\theta}-\underline{\theta}}(\overline{\theta} - \frac{p}{q})$. No matter the price strategy adopted, it can be shown that these profits are always lower than the ones obtained with a covered market configuration.

Given that we mainly refer to digital goods, we assume that the original good and its copies are identical in terms of quality. We use the expressions "pirated good" and "copy" indifferently: our definition of a copy includes both a copy made by somebody else and bought illegally[18] (*i.e.* a CD from an unauthorized vendor) and a copy made by the consumer herself (*i.e.* a CD copied with a CD burner). (Net) utility of the original good differs from that of the copy because the consumer bears different costs when purchasing the original good and when obtaining a copy. In the case of purchase (on the legal market), the cost is clearly the market price of the good. In the case of a copy, the cost c_i can represent both a cost of purchasing the good on the illegal market, which also embodies a psychological cost of not having the original good, and/or the material cost of making the copy (*i.e.* the price of the CD burner). This justifies different costs for different consumers.

Quality k is purchased rather than downloaded if and only if

$$\theta q_k - p_k \geq \theta q_k - c_i \tag{3}$$

which yields

$$p_k \leq c_i \tag{4}$$

We assume that when indifferent between pirating and purchasing a good, consumers always purchase it.

2.3 The benchmark case: no piracy

The goal of our paper is to show that the possibility of copying may induce the monopolist to differentiate its product. The issue is relevant since, as the following Proposition shows, if copying is not an available option for consumers, no vertical differentiation would arise in equilibrium.

Proposition 1 *When consumers cannot pirate the good, the pricing strategy of the monopolist entails no differentiation.*

Proof. See Appendix. ■

Given this benchmark case, in the next sections we study the impact of piracy on the profit-maximizing strategies of the monopolist. We first analyze price and quality decisions when the monopolist produces one and two qualities respectively. We then compare the profits obtained under both cases, in order to find the equilibrium strategy in terms of the number and level of the qualities produced.

[18] In order for this interpretation to work, we should assume that on the illegal market there is a large number of identical firms competing à la Bertrand and where therefore market price is equal to marginal cost.

3 The monopolist producing only one quality

We first examine the case where the monopolist produces only one quality and determine its equilibrium strategy in terms of price and quality choice.

The monopolist has three selling strategies: set such a high price that nobody buys the good ($c_L < c_H < p$), set an intermediate price that keeps only high-cost consumers in the market ($c_L < p < c_H$), or set such a low price that both types of consumers buy ($c_L < c_H < p$). Clearly, if $c_L < c_H < p$, both the high-cost and the low-cost type prefer to pirate the good rather than purchasing it (see condition 4) and the monopolist earns zero profits. We now examine the other two strategies in turn.

Lemma 2 *If $p \leq c_L < c_H$ all consumers buy the good. In equilibrium, the price is*

$$p^* = c_L \tag{5}$$

quality is

$$q^* = \frac{c_L}{\underline{\theta}} \tag{6}$$

whereas total profits are

$$\Pi^* = c_L \tag{7}$$

Proof. See appendix. ■

Lemma 3 *If $c_L < p < c_H$ only high-cost consumers buy the good. In equilibrium the price is*

$$p^{**} = c_H \tag{8}$$

quality is

$$q^{**} = \frac{c_H}{\hat{\theta}} \tag{9}$$

whereas total profits are

$$\Pi^{**} = c_H \frac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}} \tag{10}$$

Proof. See appendix. ■

By comparing equilibrium profits under the two strategies, it turns out that the optimal strategy for prices and quality depends on the degree of cost heterogeneity among consumers as described by the following proposition:

Proposition 4 *The optimal strategy entails pricing such that high-cost consumers buy and low-cost consumers pirate the good if and only if*

$$c_L < c_H \frac{\overline{\theta} - \hat{\theta}}{\overline{\theta} - \underline{\theta}} \tag{11}$$

Otherwise, the monopolist sets a price that induces both types of consumers to buy.

Proof. See Appendix. ■

The relative profitability of the two strategies depends on two elements: the relative size of the costs of pirating and the proportion of high-cost consumers. The higher c_L with respect to c_H, the more it is profitable to sell also to low-cost consumers (at price c_L). If there are many high-cost consumers it is more profitable to set a price such that only these consumers buy ($p = c_H$) rather than setting a lower price ($p = c_L$) that induces also low-cost consumers to buy because the lack of profits incurred in the low-cost segment of the market is more than compensated by the higher price the monopolist can charge in the high-cost segment. Conversely, if c_L and c_H are close enough and the proportion of high-cost consumers is small, it is better to set a low price such that both types of consumers purchase the good instead of letting low-cost pirate it.

It is immediate to check that $p^* < p^{**}$ always and $q^* < q^{**}$ iff $c_L < c_H \frac{\underline{\theta}}{\overline{\theta}}$.

4 The monopolist producing two qualities

In this section we analyze the strategy of producing two different qualities, $q_1 < q_2$. We solve backwards by first computing profits under the two alternative strategies of allowing some piracy from low-cost consumers ($p_1 < c_L < p_2 \leq c_H$) and of eliminating piracy at all ($p_1 < p_2 \leq c_L < c_H$).

As to the latter strategy, it reduces to the situation where copying is not an available option for consumers and the monopolist finds optimal not to differentiate (see Section 2.3).

Therefore, when considering differentiation, we only have to analyze the case where the monopolist chooses prices to allow piracy to exist. In this case, low-cost consumers buy quality 1 and pirate quality 2. Notice first that setting $p_1 = c_L$ would imply that low-cost consumers always prefer to pirate the high-quality good rather than buying the low-quality one (given that $\theta q_2 - c_L > \theta q_1 - c_L$): the low-quality good would never be produced and we would be back to the one-quality case. Therefore setting $p_1 = c_L$ is not an optimal strategy for the monopolist.

Among low-cost consumers, the consumer who is indifferent between purchasing quality 1 and pirating 2 has willingness to pay $\tilde{\theta}$, where $\tilde{\theta}$ solves

$$\theta q_2 - c_L = \theta q_1 - p_1$$

and is equal to

$$\tilde{\theta} = \frac{c_L - p_1}{q_2 - q_1} \tag{12}$$

The behavior of high-cost consumers is described in the following Lemma.

Lemma 5 *If $\hat{\theta} > \frac{\bar{\theta}}{2}$ all high-cost consumers with $\theta \in \left[\hat{\theta}, \bar{\theta}\right]$ buy quality q_2. If $\hat{\theta} < \frac{\bar{\theta}}{2}$, there exist a threshold $\theta^{\circ} \in \left[\hat{\theta}, \bar{\theta}\right]$, $\theta^{\circ} \equiv \frac{p_2 - p_1}{q_2 - q_1}$, such that all consumers with $\theta \in \left[\hat{\theta}, \theta^{\circ}\right]$ buy quality q_1 and those with $\theta \in \left[\theta^{\circ}, \bar{\theta}\right]$ buy quality q_2.*

Proof. *See appendix.* ■

Lemma 5 implies that when the market of high-cost consumers (whose dimension is exogenously fixed) is relatively small, it is never optimal to segment it.

We can therefore distinguish two cases

1. $\hat{\theta} > \frac{\bar{\theta}}{2}$ so that $\theta^{\circ} < \hat{\theta}$: here high-cost consumers buy quality q_2 only.
2. $\hat{\theta} < \frac{\bar{\theta}}{2}$ so that $\theta^{\circ} > \hat{\theta}$: here high-cost consumers buy both qualities.

We examine both cases separately, since they have different implications in the choice of quality levels. In particular, as we will see, the choice of q_1 in case 1 does not influence the profits obtained in the high-cost segment as it happens instead in case 2.

Demands for the two goods are defined as follows. Demand for the good of quality 1 is

$$x_1 = \frac{1}{\bar{\theta} - \underline{\theta}} \left(\tilde{\theta} - \underline{\theta} + \theta^{\circ} - \hat{\theta} \right) \tag{13}$$

since it is bought by low-cost consumers with $\theta \in \left[\underline{\theta}, \tilde{\theta}\right]$ and by high-cost consumers with $\theta \in \left[\hat{\theta}, \theta^{\circ}\right]$.

Similarly, demand for the good of quality 2 is

$$x_2 = \frac{1}{\bar{\theta} - \underline{\theta}} \left(\bar{\theta} - \theta^{\circ} \right) \tag{14}$$

Finally, piracy occurs for all consumers with $\theta \in \left[\tilde{\theta}, \hat{\theta}\right]$ and is defined as

$$x_{2P} = \frac{1}{\bar{\theta} - \underline{\theta}} \left(\hat{\theta} - \tilde{\theta} \right) \tag{15}$$

We now examine the two cases illustrated by Lemma 5 separately, starting from the case with $\hat{\theta} > \frac{\bar{\theta}}{2}$, where high-cost consumers buy the high quality good only.

4.1 High-cost consumers buy quality q_2 only

In this case, high-cost consumers buy quality q_2 only. Low-cost consumers with $\theta < \tilde{\theta}$ buy q_1 and those with $\theta > \tilde{\theta}$ pirate q_2.

The equilibrium strategy for prices and quality is described by the following lemma:

Lemma 6 *When $\hat{\theta} > \frac{\bar{\theta}}{2}$, the monopolist chooses (q_1, q_2) such that the demand for good 1 is maximized and therefore piracy does not occur in equilibrium. Equilibrium prices and qualities are respectively*

$$p_1^{diff} = \frac{c_L \left(\hat{\theta} - \underline{\theta} \right)}{2\hat{\theta} - \underline{\theta}}, p_2^{diff} = c_H \tag{16}$$

and

$$q_2^{diff} = \frac{c_H}{\hat{\theta}}, q_1^{diff} = \frac{c_H \left(2\hat{\theta} - \underline{\theta} \right) - c_L \hat{\theta}}{\hat{\theta} \left(2\hat{\theta} - \underline{\theta} \right)} \tag{17}$$

Equilibrium profits are

$$\Pi^{diff} = \frac{c_L \left(\hat{\theta} - \underline{\theta} \right)^2 + c_H \left(\bar{\theta} - \hat{\theta} \right) \left(2\hat{\theta} - \underline{\theta} \right)}{\left(\bar{\theta} - \underline{\theta} \right) \left(2\hat{\theta} - \underline{\theta} \right)} \tag{18}$$

Proof. See Appendix. ■

It can be readily verified that q_1^{diff} is increasing in c_H and decreasing in c_L, whereas q_2^{diff} is increasing in c_H. q_1^{diff} is decreasing in c_L because the lower the cost of pirating, the higher the quality the monopolist has to offer to low-cost consumers to induce them to buy the good. Profits in (18) are always positive.

In this differentiating equilibrium, the monopolist eliminates piracy not through low prices (i.e. setting $p_1 < p_2 \leq c_L < c_H$) but through an appropriate choice of quality levels. This is done by fixing such a high level of q_1 that all low-cost consumers decide to buy it.

4.2 High-cost consumers buy both qualities

In the previous section, profits from selling in the high-cost segment of the market do not depend on q_1 and this clearly influences its equilibrium level. In particular, q_1 positively affects the demand of low-cost consumers while not having any impact on the demand of high-cost consumers (which depends only on q_2). When instead $\hat{\theta} < \frac{\bar{\theta}}{2}$, i.e. the proportion of high-cost consumers is high, the impact of $q_1$on profits is ambiguous. In this case, high-cost consumers buy both qualities, so that an increase in q_1 decreases profits in this segment through its influence on p_1. If p_1 is low enough, high-cost consumers may in fact decide to buy q_1 rather than q_2 (more profitable for the monopolist due to $p_2 > p_1$). In particular, the more the two quality levels are close to each other, the lower the prices that can be set for the two goods, and therefore the lower the profits that can be extracted from the high-cost segment of the market. Therefore, q_1 is not necessarily set so high that the demand for the pirated good is zero and in the equilibrium we might experience both differentiation and piracy.

In this case, high-cost consumers with $\theta < \theta^{\circ}$ buy quality q_1, while those with $\theta > \theta^{\circ}$ buy quality q_2. As before, low-cost consumers with $\theta < \tilde{\theta}$ buy q_1 and those with $\theta > \tilde{\theta}$ pirate q_2.

The equilibrium strategy for prices and quality is described by the following lemma, where $f \equiv (q_2 - q_1)$. In what follows, the absolute levels of q_1 and q_2 are irrelevant and only their difference matters.[19]

Lemma 7 *When $\hat{\theta} < \frac{\bar{\theta}}{2}$, the monopolist always chooses either maximum or minimum differentiation.*

1. *With minimum differentiation, qualities are chosen such that their difference is $f_{\min} = \frac{c_L}{\bar{\theta}+\hat{\theta}-\underline{\theta}}$ and profits are*

$$\Pi_{\min} = \frac{c_L \left[5\bar{\theta}^2 - 4\underline{\theta}\left(2\bar{\theta} - \underline{\theta}\right)\right]}{4\left(\bar{\theta} - \underline{\theta}\right)\left(\bar{\theta} + \hat{\theta} - \underline{\theta}\right)} \tag{19}$$

   Here, x_1 is set as high as possible and no piracy arises, i.e. $x_{2P} = 0$.

2. *With maximum differentiation, qualities are chosen such that their dif-*

[19]This is a typical feature of models of vertical differentiation assuming a covered market and no costs of quality improvement.

ference is $f_{\max} = \frac{2c_H - c_L}{2\bar{\theta} - \hat{\theta} - \underline{\theta}}$ *and profits are*

$$\Pi_{\max} = \frac{\bar{\theta}^2 c_L^2 - 2\bar{\theta} c_H c_L \left(\hat{\theta} + \underline{\theta}\right) + 2c_H^2 \left[2\bar{\theta}^2 - 2\bar{\theta}\left(\hat{\theta} + \underline{\theta}\right) + \left(\hat{\theta} + \underline{\theta}\right)^2\right]}{2\left(2c_H - c_L\right)\left(\bar{\theta} - \underline{\theta}\right)\left(2\bar{\theta} - \underline{\theta} - \hat{\theta}\right)} \tag{20}$$

In this case, demand for piracy is

$$x_{2P} = \frac{2c_H\left(\bar{\theta} + \hat{\theta} - \underline{\theta}\right) - c_L\left(3\bar{\theta} - 2\underline{\theta}\right)}{2\left(2c_H - c_L\right)} \tag{21}$$

which is positive if the condition

$$c_L < c_H \frac{2\left(\bar{\theta} + \hat{\theta} - \underline{\theta}\right)}{3\bar{\theta} - 2\underline{\theta}} \tag{22}$$

holds.

Proof. See Appendix. ■

It is interesting to notice that condition (22) in Lemma 7 implies $f_{\max} > f_{\min}$ because, with $f = f_{\min}$, $x_{2P} = 0$ by definition of $f_{\min}$. Notice also, from (21) that $\frac{\partial x_{2P}}{\partial c_L} < 0$, that is the extent of the piracy allowed by the monopolist decreases with c_L. This happens because with a high c_L the monopolist is able to charge a higher price for the low quality without violating the constraint $p_1 \leq c_L$ and is therefore able to produce a higher quality q_1, therefore diverting more consumers from the pirated good.

The monopolist will choose either minimum or maximum differentiation according to the parameters' constellation. In any case, we must make sure that two conditions are always satisfied. Specifically, quality q_2 must be sufficiently higher than q_1, so that $\tilde{\theta} \leq \hat{\theta}$, i.e. $x_{2P} \geq 0$ (the demand for the pirated good is non negative). In addition, we require q_1 to be such that its demand by low-cost consumers is non negative too (that is $\underline{\theta} < \tilde{\theta}$). Both conditions are met iff:

1. $c_L \leq \frac{2c_H\left(\bar{\theta}+\hat{\theta}-\underline{\theta}\right)}{3\bar{\theta}-2\underline{\theta}}$ to guarantee that $x_{2P} \geq 0$ (condition (22) in Lemma 7)

2. $c_L > \frac{2c_H\left(\bar{\theta}-\hat{\theta}+\underline{\theta}\right)}{3\bar{\theta}-2\hat{\theta}}$ to ensure that $\tilde{\theta} > \underline{\theta}$.

Which, given that $\frac{2c_H\left(\bar{\theta}-\hat{\theta}+\underline{\theta}\right)}{3\bar{\theta}-2\hat{\theta}} < \frac{2c_H\left(\bar{\theta}+\hat{\theta}-\underline{\theta}\right)}{3\bar{\theta}-2\underline{\theta}}$ always when $\bar{\theta} > 2\hat{\theta}$, can be rewritten as a unique condition

$$\frac{2c_H\left(\bar{\theta}-\hat{\theta}+\underline{\theta}\right)}{3\bar{\theta}-2\hat{\theta}} < c_L \leq \frac{2c_H\left(\bar{\theta}+\hat{\theta}-\underline{\theta}\right)}{3\bar{\theta}-2\underline{\theta}} \tag{23}$$

As we will need to refer to condition (23) several times in what follows, we define

$$\underline{c}_L = \frac{2c_H\left(\bar{\theta}-\hat{\theta}+\underline{\theta}\right)}{3\bar{\theta}-2\hat{\theta}} \tag{24}$$

$$\bar{c}_L = \frac{2c_H\left(\bar{\theta}+\hat{\theta}-\underline{\theta}\right)}{3\bar{\theta}-2\underline{\theta}} \tag{25}$$

The following proposition establishes the conditions such that maximum differentiation yields higher profits than minimum differentiation:

Proposition 8 *When $\hat{\theta} < \frac{\bar{\theta}}{2}$ and the monopolist differentiates its product, there exists a value for $\hat{\theta}$,*

$$\hat{\theta}_1 = \frac{\bar{\theta}^2}{4\left(\bar{\theta}-\underline{\theta}\right)} \tag{26}$$

such that:

1. *If $\hat{\theta} < \hat{\theta}_1$ maximum differentiation yields overall higher profits than minimum differentiation.*
2. *If $\hat{\theta} > \hat{\theta}_1$ there exists a value for c_L, $\tilde{c}_{L1}$, such that minimum differentiation yields higher profits for all $\tilde{c}_{L1} < c_L < \bar{c}_L$ and maximum differentiation yields higher profits for $\underline{c}_L < c_L < \tilde{c}_{L1}$.*

Proof. See Appendix. ■

The intuition for this result can be given as follows. When $\hat{\theta}$ is low (the situation depicted in Figure 1) the high-cost segment of the market, consisting of customers with $\theta \in \left[\hat{\theta}, \bar{\theta}\right]$, is large. Therefore, the monopolist would rather produce highly differentiated quality levels to be sold at high prices (notice that $\frac{\partial p_i}{\partial f} > 0$, that is prices depend positively on the difference in quality levels) than selling at lower prices qualities that are close substitutes.

In other words, profits from selling at high prices[20] on the high segment of the market more than compensate the loss of customers due to piracy in the segment $\left[\tilde{\theta},\hat{\theta}\right]$.

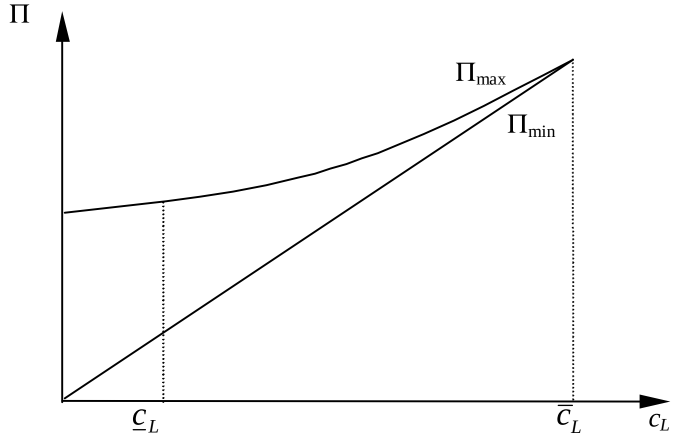

Figure 1

Conversely, when $\hat{\theta}$ is high (the situation depicted in Figure 2), the high-quality segment of the market is small and the profit accruing from it little. The monopolist prefers maximum differentiation only when c_L is very low. This happens because, when c_L is close to zero, $\Pi_{\min}$ is close to zero too (both prices and the difference between qualities, $f_{\min}$ tend to zero) whereas with maximum differentiation prices are both high and profits from the high segment of the market high too notwithstanding the high number of low-cost consumers pirating the good.[21]

[20] The price for the high quality when the monopolist applies maximum differentiation, $p_2 = c_H$, is higher than the price for the high quality with minimum differentiation. Therefore, profits from selling to the high-cost segment of the market (consumers with $\theta \in \left[\frac{\bar{\theta}}{2},\bar{\theta}\right]$) are higher with maximum differentiation. The same applies to p_1.

[21] It is possible to show that demand for the low-quality good from the low-cost consumers (demand defined as $x_1^L = \frac{\tilde{\theta}-\underline{\theta}}{\theta-\underline{\theta}}$) is increasing in c_L.

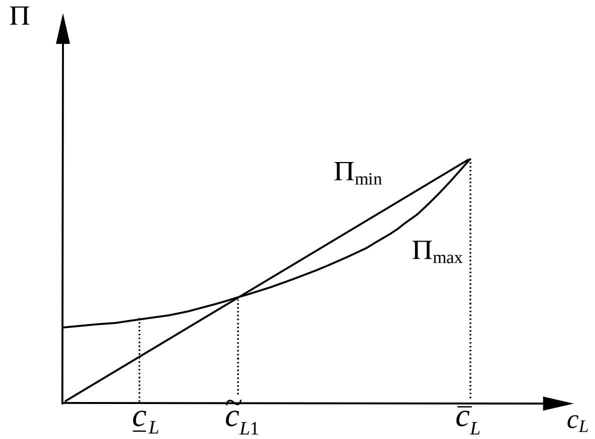

Figure 2

Next section compares all strategies available to the monopolist, with the aim to establish when product differentiation is an equilibrium strategy and the conditions under which piracy occurs in the market.

5 One or two qualities?

In the first stage, the monopolist chooses whether to produce one or two qualities. In order to make this decision, it compares profits from each available alternative. The main finding is that piracy may induce the monopolist to produce different qualities even when there are no production costs. In fact, the existence of piracy itself may justify the introduction of lower qualities, in the attempt to move away consumers with lower willingness to pay from the illegal market, where copies of the high quality good are exchanged, and let them buy the low quality instead. When the proportion of high-cost consumers is low, in equilibrium the monopolist may produce a low quality that is high enough to be appealing to all consumers: this will eliminate demand for copies and hence piracy. Piracy is however observed in many markets. In our model, the presence of piracy may arise in a differentiating equilibrium when the proportion of high-cost consumers is high.

5.1 High-cost consumers buy quality q_2 only

When $\hat{\theta} > \frac{\bar{\theta}}{2}$, in equilibrium the monopolist produces two quality levels only if this yields higher profits than producing one quality only (by setting prices such that piracy is either eliminated or tolerated).

The following proposition indicates the conditions under which producing two qualities is the equilibrium strategy.

Proposition 9 *In equilibrium, the monopolist chooses to produce two quality levels if* $c_L < c_H \frac{(\bar{\theta}-\hat{\theta})(2\hat{\theta}-\underline{\theta})}{\bar{\theta}(2\hat{\theta}-\underline{\theta})-\hat{\theta}^2}$. *Otherwise, it is more profitable to produce one quality. In both cases, prices and quality levels are set such that no piracy arises.*

Proof. See Appendix. ■

The intuition for this result can be given by Figure 3 below. The profit when the monopolist differentiates, Π_{diff}, is always above Π^{**} (the profit with one quality only and piracy) but for the case where $c_L = 0$. This can be seen immediately by rewriting Π_{diff} in expression (18) as follows

$$\Pi^{diff} = c_H \frac{\bar{\theta}-\hat{\theta}}{\bar{\theta}-\underline{\theta}} + c_L \frac{\hat{\theta}-\underline{\theta}}{2\hat{\theta}-\underline{\theta}} = \Pi^{**} + c_L \frac{\hat{\theta}-\underline{\theta}}{2\hat{\theta}-\underline{\theta}} \tag{27}$$

Producing one quality while allowing some piracy is never an equilibrium strategy because the monopolist could always profitably introduce another (lower) quality which would attract at least some of the low-cost consumers that would have otherwise pirated the good. For very high levels of c_L (close to c_H) the monopolist prefers to produce one quality because a high price $p \leq c_L$ can be set and it is then profitable to sell the same (high) quality to the whole market. If c_L is low and c_H is much higher than c_L, a strategy such that $p_1 < c_L < p_2 \leq c_H$ is more profitable, where the monopolist sells to the high-cost segment of the market at a very high price and captures the low-cost segment by selling a low-price, low-quality good.[22]

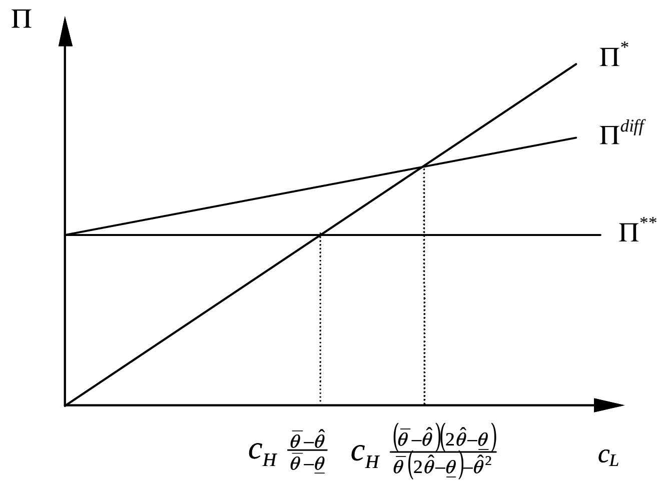

Figure 3

[22]It is immediate to check that Π^{diff} crosses $\Pi^* = c_L$ for $c_L > c_H \frac{\bar{\theta}-\hat{\theta}}{\bar{\theta}-\underline{\theta}}$.

5.2 High-cost consumers buy both qualities

We now proceed by showing the existence of an equilibrium with maximum differentiation (as well as equilibria with one quality only).

We obtain the following results.

Proposition 10 *There exist one value for c_L, c_L^{pir}, such that*

1. *If $\hat{\theta} < \hat{\theta}_1$ the following cases arise:*
   - *(a) if $\bar{\theta} > \left(3+\sqrt{5}\right)\underline{\theta}$ then: if $\hat{\theta}_2 < \hat{\theta} < \hat{\theta}_1$ the monopolist chooses maximum differentiation for all $c_L^{pir} < c_L < \bar{c}_L$ and produces one quality allowing piracy if $c_L < c_L^{pir}$; if $\hat{\theta} < \hat{\theta}_2 < \hat{\theta}_1$ then the monopolist always produces one quality allowing piracy;*
   - *(b) if $\bar{\theta} < \left(3+\sqrt{5}\right)\underline{\theta}$ the monopolist always produces one quality allowing piracy.*
2. *If $\hat{\theta} > \hat{\theta}_1$ the monopolist produces one quality, either by letting low-cost consumers pirate the good (if $c_L < c_H\frac{\bar{\theta}-\hat{\theta}}{\bar{\theta}-\underline{\theta}}$) or by setting a price that induces both types of consumers to buy (if $c_L > c_H\frac{\bar{\theta}-\hat{\theta}}{\bar{\theta}-\underline{\theta}}$).*

Proof. See Appendix. ■

The intuition for the results in Proposition 10 can be given as follows. From Proposition 8 we know that profits are higher with maximum rather than minimum differentiation when $\hat{\theta} < \hat{\theta}_1$. However, it can also be shown that in this case minimum differentiation is better than selling one quality to the whole market. Therefore it follows that maximum differentiation is preferred to selling a single quality to the whole market. This latter result follows from the much higher profits the monopolist can extract from the high-cost, high-quality segment of the market $\left[\frac{\bar{\theta}}{2},\bar{\theta}\right]$ (which now is large), where q_2 can be sold at $p_2 = c_H$ instead of $p = c_L$.

The relevant comparison when $\hat{\theta} < \hat{\theta}_1$ is therefore between $\Pi_{\max}$ and Π^{**}, the curve and the straight line parallel to the $x-$axis in Figure 4 below. It is then immediate to check that $\Pi^{**} > \Pi_{\max}$ for all $\underline{c}_L < c_L < c_L^{pir}$. Then, for low c_L, selling one quality to the high-cost segment of the market is a better strategy than selling two qualities to that segment of the market, while selling the lower quality to a share $\frac{\tilde{\theta}-\underline{\theta}}{\bar{\theta}-\underline{\theta}}$ of the low-cost segment and letting all other consumers pirate q_2. When c_L is low, the extra profit the monopolist makes by selling q_1 to the consumers with $\theta \in \left[\underline{\theta},\tilde{\theta}\right]$ is very low (recall that p_1 is positively related to c_L). The loss of profits from selling q_2 only to a share $\frac{\bar{\theta}-\frac{\bar{\theta}}{2}}{\bar{\theta}-\underline{\theta}}$

of the market and q_1 at p_1 to the remaining share is instead quite substantial. That explains why $\Pi^{**} > \Pi_{\max}$ for low c_L whereas the order is reversed for sufficiently high values of c_L.

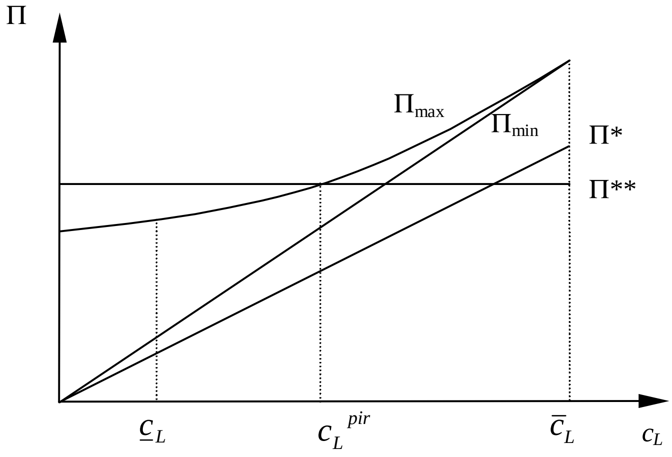

Figure 4

A less interesting case arises when consumers are enough homogeneous in their taste for quality ($\bar{\theta} < \left(3+\sqrt{5}\right)\underline{\theta}$). In this case the monopolist always finds more profitable to sell a single quality to high-cost consumers only (whose proportion is relatively high given that $\hat{\theta} < \hat{\theta}_1$) rather than differentiating its product.

When $\hat{\theta} > \hat{\theta}_1$ and the high-cost segment of the market smaller than in the first part of Proposition 10, selling one quality to the whole market is overall better than minimum differentiation. The relevant comparison therefore is between $\Pi_{\max}$, Π^{**} and Π^{*}. However, it turns out that in the relevant range of the parameters, maximum differentiation is always dominated by the strategy of selling a single quality to the whole market. Maximum differentiation yields lower profits that selling one quality to the whole market because the loss from piracy from consumers with $\theta \in \left[\tilde{\theta}, \hat{\theta}\right]$ is not compensated by the higher profits from selling a higher quality at $p_2 = c_H$ in the segment $\left[\frac{\bar{\theta}}{2}, \bar{\theta}\right]$ of the market, as such segment is too small. The relevant comparison is therefore between the strategy of selling a single quality to the whole market and the strategy of letting low-cost consumers pirate the good, as in Proposition 4. This case is depicted in Figure 5 below.

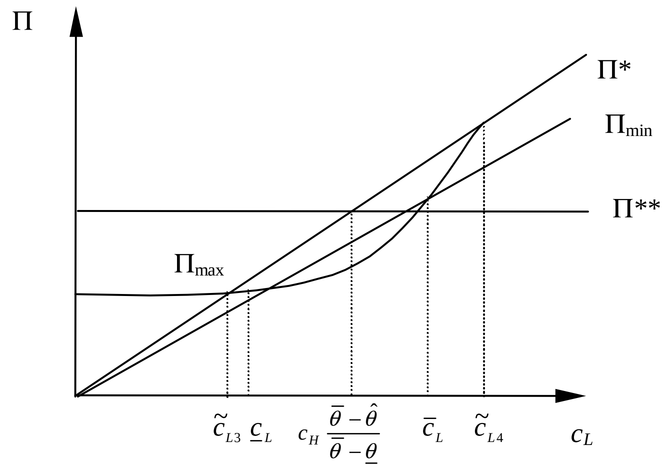

Figure 5

6 Conclusions

We have analyzed the choice of quality levels by a monopolist in a market where piracy exists. The main idea is that the introduction of a lower quality may be a device through which the monopolist manages to capture some consumers that would otherwise prefer to pirate the good. We have shown that there are ranges of the parameters for which the monopolist prefers to produce two qualities rather than one quality only. The relative profitability of these two strategies essentially depends on the degree of consumers' heterogeneity in the cost of pirating and on the relative proportion of the two types of consumers (those with low cost of pirating and those with high cost of pirating). In particular, we have showed that, when the proportion of high-cost consumers is low and differentiation is the optimal strategy, the quality level chosen for the low-quality good is such that piracy is completely eliminated in equilibrium.

When instead the proportion of high-cost consumers is high, there is an equilibrium with maximum differentiation where the monopolist admits some piracy. This happens because, since in this case the monopolist finds profitable to sell both qualities to the high-cost segment of the market, the demand of the high-quality good depends positively on p_1. In particular, the more the two quality levels are close to each others, the lower the prices that can be set for the two goods, and therefore the lower the profits that can be extracted from the high-cost segment of the market. Therefore, the low-quality is not necessarily set so high that the demand for the pirated good is zero and in the equilibrium we might experience both differentiation and piracy.

An interesting extension would be to evaluate the impact of more restrictive copyright laws on equilibrium prices and qualities in a market where consumers can pirate the good. We can do so by incorporating in the cost of copying the expected fine that the consumer has to pay if caught in possession of the illegal good and study how the monopolist's strategies would respond to an increase in the expected fine. We expect that an increased copyright protection, by making pirating more costly and therefore less viable for consumers, would reduce the monopolist's incentive to produce a second (lower) quality in order to reduce piracy. The degree of copyright enforcement could also be seen as an endogenous choice of the monopolist by appropriately specifying its objective function.

A Appendix: Proofs

Proof of Proposition 1 When two quality levels are produced in a "covered market" configuration, the demands for quality 1 and quality 2 are $x_1 = \frac{\theta^\circ - \underline{\theta}}{\bar{\theta} - \underline{\theta}}$, $x_2 = \frac{\bar{\theta} - \theta^\circ}{\bar{\theta} - \underline{\theta}}$ respectively, where $\theta^\circ = \frac{p_2 - p_1}{q_2 - q_1}$ is the preference for quality of the marginal consumer who is indifferent between buying quality 1 and quality 2. Profits are

$$\Pi(p_1, p_2) = \frac{1}{\bar{\theta} - \underline{\theta}} \left[p_1 \left(\frac{p_2 - p_1}{q_2 - q_1} - \underline{\theta} \right) + p_2 \left(\bar{\theta} - \frac{p_2 - p_1}{q_2 - q_1} \right) \right], \tag{28}$$

Following the three- stage decisional process described above, the monopolist will first define its optimal price strategy. Note however that $x_1 \geq 0$ implies $\frac{\partial \Pi}{\partial p_1} > 0$, $\forall p_2 > 0$. The monopolist would then find optimal to set p_1 as high as possible, i.e. such that $x_1 = 0$. This implies $p_1 = p_2 - \underline{\theta}(q_2 - q_1)$ and $x_2 = 1$. Therefore, in the quality stage the monopolist would choose to produce a single quality. ■

Proof of Lemma 2 When $p \leq c_L < c_H$, profits are $\Pi(p) = p$ since at this price all consumers buy the good. The monopolist will set the highest possible price subject to the constraint that the consumer with the lowest willingness to pay ($\underline{\theta}$) has non-negative utility, that is

$$p = \underline{\theta} q$$

Total profits are therefore increasing in q. The highest q the monopolist can produce is the one such that $p^* = c_L$:

$$q^* = \frac{p}{\underline{\theta}} = \frac{c_L}{\underline{\theta}} \tag{29}$$

Equilibrium profits are

$$\Pi^* = c_L \tag{30}$$

■

Proof of Lemma 3 When $c_L < p \leq c_H$, high-cost consumers buy the good and low-cost ones pirate it. Demand is therefore given by the high-cost fraction of consumers

$$x = \frac{1}{\overline{\theta} - \underline{\theta}}[\overline{\theta} - \hat{\theta}] \tag{31}$$

and profits are

$$\Pi(p) = \frac{p}{\overline{\theta} - \underline{\theta}}[\overline{\theta} - \hat{\theta}] \tag{32}$$

Since profits are increasing in p, the monopolist will choose the highest price such that the marginal buyer (the one with $\theta = \hat{\theta}$) has zero surplus, i.e. $p = \hat{\theta}q$. Plugging this into (32), we get

$$\Pi(q) = \hat{\theta}q\frac{\overline{\theta} - \hat{\theta}}{\overline{\theta} - \underline{\theta}} \tag{33}$$

Profits are increasing in the quality of the good and the monopolist will set q as high as possible given the constraint that $p \leq c_H$, i.e. $q \leq \frac{c_H}{\hat{\theta}}$. Hence, the equilibrium price and quality are

$$p^{**} = c_H \tag{34}$$

$$q^{**} = \frac{c_H}{\hat{\theta}} \tag{35}$$

The equilibrium profits are

$$\Pi^{**} = c_H\frac{\overline{\theta} - \hat{\theta}}{\overline{\theta} - \underline{\theta}} \tag{36}$$

■

Proof of Proposition 4 The optimal strategy for the monopolist is determined by comparing the equilibrium profits under the three strategies. Since the first strategy, i.e. setting such a high price that nobody purchases the good, leads to zero profits and is never optimal, the relevant strategies are only the second and the third ones. It turns out that $\Pi^* > \Pi^{**}$ (*i.e.* it is better to set a price that induces both types of consumers to buy ($p = c_L$) if and only if

$$c_L > c_H \frac{\overline{\theta} - \hat{\theta}}{\overline{\theta} - \underline{\theta}} \tag{37}$$

■

Proof of Lemma 5 High-cost consumers can either buy 1 or buy 2, and they buy 1 iff

$$\theta < \theta^{\circ} \equiv \frac{p_2 - p_1}{q_2 - q_1}$$

Hence, demand for the good of quality 1 is

$$x_1 = \frac{1}{\overline{\theta} - \underline{\theta}} \left(\tilde{\theta} - \underline{\theta} + \theta^{\circ} - \hat{\theta} \right) \tag{38}$$

Similarly, demand for the good of quality 2 is

$$x_2 = \frac{1}{\overline{\theta} - \underline{\theta}} \left(\overline{\theta} - \theta^{\circ} \right) \tag{39}$$

The monopolist's profits thus are

$$\Pi(p_1, p_2) = \frac{1}{\overline{\theta} - \underline{\theta}} \left[p_1 \left(\frac{c_L + p_2 - 2p_1}{q_2 - q_1} - \underline{\theta} - \hat{\theta} \right) + p_2 \left(\overline{\theta} - \frac{p_2 - p_1}{q_2 - q_1} \right) \right] \tag{40}$$

The profit function is globally concave in (p_1, p_2) and the first-order condition $\frac{\partial \Pi(.)}{\partial p_2} = 0$ gives $p_2 = p_1 + \frac{\overline{\theta}}{2}(q_2 - q_1)$, which implies $\theta^{\circ} = \frac{\overline{\theta}}{2}$. Hence, high cost consumers buy the low quality good iff $\hat{\theta} < \frac{\overline{\theta}}{2}$. ■

Proof of Lemma 6 Given the demands in expression (13), (14), the monopolist's profits are

$$\Pi(p_1, p_2) = \frac{1}{\overline{\theta} - \underline{\theta}} \left[p_1 \left(\frac{c_L - p_1}{q_2 - q_1} - \underline{\theta} \right) + p_2 \left(\overline{\theta} - \hat{\theta} \right) \right] \tag{41}$$

Profits are always increasing in p_2. The monopolist fixes the highest p_2 that makes all high-cost consumers purchase the good, i.e. such that the last consumer purchasing quality 2 has zero surplus. Hence $p_2 = \hat{\theta} q_2$.

From the first order condition on p_1

$$p_1 = \frac{c_L - \underline{\theta}(q_2 - q_1)}{2} \tag{42}$$

Notice that $p_1 > 0$ if and only if $c_L > \underline{\theta}\,(q_2 - q_1)$. The second-order condition is satisfied since $\frac{\partial^2 \Pi}{\partial p_1^2} = -2$. We now solve backward and obtain the optimal quality levels. Substituting p_1 and p_2 into (41), the monopolist's profit becomes

$$\Pi(q_1, q_2) = \frac{c_L^2 - 2c_L(q_2 - q_1)\underline{\theta} + (q_2 - q_1)\left[\underline{\theta}^2(q_2 - q_1) + 4\hat{\theta}\left(\bar{\theta} - \hat{\theta}\right)q_2\right]}{4\left(\bar{\theta} - \underline{\theta}\right)(q_2 - q_1)} \tag{43}$$

Notice that $\frac{\partial \Pi(.)}{\partial q_1} = \frac{c_L^2 - (q_2 - q_1)^2 \underline{\theta}^2}{4\left(\bar{\theta} - \underline{\theta}\right)(q_2 - q_1)^2} > 0$ whenever $p_1 > 0$. The monopolist thus sets q_1 as high as possible. This is done by choosing it such that x_1 is maximized and piracy disappears, i.e. $x_{2P} = 0$. Hence q_1 is set so that $\tilde{\theta} = \hat{\theta}$:

$$q_1^{diff} = q_2 - \frac{c_L}{2\hat{\theta} - \underline{\theta}} \tag{44}$$

This means that the last consumer pirating the good (*i.e.* $\tilde{\theta}$) is made indifferent between buying and pirating good 2. Subject to the constraint that $q_1 = q_1^{diff}$ profits are now equal to:

$$\Pi(q_2) = \frac{c_L\left(\hat{\theta} - \underline{\theta}\right)^2 + q_2\left(\bar{\theta} - \hat{\theta}\right)\hat{\theta}\left(2\hat{\theta} - \underline{\theta}\right)}{\left(\bar{\theta} - \underline{\theta}\right)\left(2\hat{\theta} - \underline{\theta}\right)} \tag{45}$$

Again, $\frac{\partial \Pi(q_2)}{\partial q_2} > 0$ and q_2 is set at its highest possible level. Since prices are constrained by the condition $p_1 < c_L < p_2 \leq c_H$ and since $p_2 = \hat{\theta} q_2$, the maximum q_2 has to satisfy $\hat{\theta} q_2 = c_H$. Thus the equilibrium level of q_2 is

$$q_2^{diff} = \frac{c_H}{\hat{\theta}} \tag{46}$$

Substituting (46) into (44), q_1^{diff} is equal to

$$q_1^{diff} = \frac{c_H\left(2\hat{\theta} - \underline{\theta}\right) - c_L\hat{\theta}}{\hat{\theta}\left(2\hat{\theta} - \underline{\theta}\right)} \tag{47}$$

Further substitutions of (q_1^{diff}, q_2^{diff}) into (42) p_2 and (45) yields p_1^{diff}, p_2^{diff}, Π^{diff} as written in the lemma's statement. It is immediate to check that $p_1^{diff} < c_L < p_2^{diff}$ and $q_2^{diff} > q_1^{diff} > 0$ (As $q_2^{diff} - q_1^{diff} = \frac{c_L}{2\hat{\theta}-\underline{\theta}} > 0$) as required. Also, profits are positive for any value of the parameters. ■

Proof of Lemma 7 Demand for quality q_2 is now

$$x_2 = \frac{1}{\bar{\theta}-\underline{\theta}}\left[\bar{\theta}-\theta^{\circ}\right] \tag{48}$$

while demand for quality q_1 is

$$x_1 = \frac{1}{\bar{\theta}-\underline{\theta}}\left[\left(\theta^{\circ}-\hat{\theta}\right)+\left(\tilde{\theta}-\underline{\theta}\right)\right] \tag{49}$$

The demand for the pirated good of quality 2 is again

$$x_{2P} = \frac{1}{\bar{\theta}-\underline{\theta}}[\hat{\theta}-\tilde{\theta}]$$

The monopolist's profits thus are

$$\Pi(p_1,p_2) = \frac{1}{\bar{\theta}-\underline{\theta}}\left\{p_1\left(\frac{c_L+p_2-2p_1}{q_2-q_1}-\hat{\theta}-\underline{\theta}\right)+p_2\left(\bar{\theta}-\frac{p_2-p_1}{q_2-q_1}\right)\right\} \tag{50}$$

The first order conditions with respect to p_1 and p_2 are respectively

$$\frac{\partial\Pi(.)}{\partial p_1} = \frac{c_L+2\left(p_2-2p_1\right)-\left(\hat{\theta}-\underline{\theta}\right)\left(q_2-q_1\right)}{\left(\bar{\theta}-\underline{\theta}\right)\left(q_2-q_1\right)} = 0 \tag{51}$$

$$\frac{\partial\Pi(.)}{\partial p_2} = \frac{\bar{\theta}\left(q_2-q_1\right)-2\left(p_2-p_1\right)}{\left(\bar{\theta}-\underline{\theta}\right)\left(q_2-q_1\right)} = 0 \tag{52}$$

It is immediate to check that the second order conditions are satisfied. Solving for the two prices we obtain

$$p_1 = \frac{c_L+f\left(\bar{\theta}-\hat{\theta}-\underline{\theta}\right)}{2}, \tag{53}$$

$$p_2 = \frac{c_L+f\left(2\bar{\theta}-\hat{\theta}-\underline{\theta}\right)}{2} \tag{54}$$

In the quality stage, the demands for q_1 and q_2 by high-cost consumers are $\frac{\bar{\theta}-2\hat{\theta}}{2(\bar{\theta}-\underline{\theta})}$ and $\frac{\bar{\theta}}{2(\bar{\theta}-\underline{\theta})}$ respectively. Notice that they are independent of f. Profits can be rewritten as

$$\Pi(f)=\frac{c_L^2+2c_Lf\left(\bar{\theta}-\hat{\theta}-\underline{\theta}\right)+f^2\left[\bar{\theta}^2+\left(\bar{\theta}-\left(\hat{\theta}+\underline{\theta}\right)\right)^2\right]}{4\left(\bar{\theta}-\underline{\theta}\right)f} \tag{55}$$

It can be noticed that profits depend exclusively on f and that

$$\frac{d\Pi(f)}{df}=\frac{1}{4}\frac{f^2\left[(\bar{\theta}-\hat{\theta})^2+(\bar{\theta}-\underline{\theta})^2+2\hat{\theta}\underline{\theta}\right]-c_L^2}{\left(\bar{\theta}-\underline{\theta}\right)f^2}$$

$$\frac{d^2\Pi(f)}{df^2}=\frac{c_L^2}{2\left(\bar{\theta}-\underline{\theta}\right)f^3}>0$$

The profit function is convex and can be either decreasing or U-shaped in f depending on the value of the parameters. Any critical point is therefore a minimum and optimal solutions for f are to be found in the extremes, with the monopolist choosing either maximum or minimum differentiation for the quality levels.

Minimum Differentiation The profit maximizing choice of the monopolist will be minimum differentiation (f low) either when $\Pi(f)$ is decreasing or when $\Pi(f)$ is U-shaped and profits from setting qualities as close as possible are higher than profits with maximum differentiation. We must have that the demand for piracy is non-negative. According to (15), $x_{2P}\geq 0$ iff $\hat{\theta}-\tilde{\theta}\geq 0$, and after substituting p_1 from (53) into (15), the smallest value for $f=q_2-q_1$ (that makes $\tilde{\theta}=\hat{\theta}$) is

$$f_{\min}=\frac{c_L}{\bar{\theta}+\hat{\theta}-\underline{\theta}} \tag{56}$$

Since $f_{\min}>0$, $q_2>q_1$ and, from (55), total profits are

$$\Pi_{\min}=\frac{c_L\left[5\bar{\theta}^2-4\underline{\theta}\left(2\bar{\theta}-\underline{\theta}\right)\right]}{4\left(\bar{\theta}-\underline{\theta}\right)\left(\bar{\theta}+\hat{\theta}-\underline{\theta}\right)} \tag{57}$$

which are always positive.

Maximum Differentiation With maximum differentiation the monopolist sets quality q_2 at the highest possible level, whereas quality q_1 is set as low as possible. Given that p_2 in (54) is increasing in q_2, quality q_2 is set so that $p_2 = c_H$. As to q_1, it is chosen such that the utility of the consumer with the lowest willingness to pay is zero, i.e.

$$\underline{\theta} q_1 - p_1 = 0 \tag{58}$$

Substituting p_1 from (53) into (58) and setting p_2 from (54) equal to c_H, we then have a system whose solution yields the levels $q_1^{\max}$ and $q_2^{\max}$ chosen with maximum differentiation

$$\underline{\theta} q_1 - \frac{c_L + (q_2 - q_1)\left(\bar{\theta} - \hat{\theta} - \underline{\theta}\right)}{2} = 0 \tag{59}$$

$$c_H - \frac{c_L + (q_2 - q_1)\left(2\bar{\theta} - \hat{\theta} - \underline{\theta}\right)}{2} = 0 \tag{60}$$

The solution is

$$q_1^{\max} = \frac{\bar{\theta} c_L + 2c_H\left(\bar{\theta} - \hat{\theta} - \underline{\theta}\right)}{2\underline{\theta}\left(2\bar{\theta} - \hat{\theta} - \underline{\theta}\right)} \tag{61}$$

$$q_2^{\max} = \frac{c_L\left(\bar{\theta} - 2\underline{\theta}\right) + 2c_H\left(\bar{\theta} - \hat{\theta} + \underline{\theta}\right)}{2\underline{\theta}\left(2\bar{\theta} - \hat{\theta} - \underline{\theta}\right)} \tag{62}$$

and

$$f_{\max} = \frac{2c_H - c_L}{2\bar{\theta} - \hat{\theta} - \underline{\theta}} > 0 \tag{63}$$

Prices are

$$p_1 = \frac{c_L \bar{\theta} + 2c_H\left(\bar{\theta} - \hat{\theta} - \underline{\theta}\right)}{2\left(2\bar{\theta} - \hat{\theta} - \underline{\theta}\right)} \tag{64}$$

$$p_2 = c_H \tag{65}$$

From (55), total profits are

$$\Pi^{\max} = \frac{\bar{\theta}^2 c_L^2 - 2\bar{\theta} c_H c_L\left(\hat{\theta} + \underline{\theta}\right) + 2c_H^2\left[2\bar{\theta}^2 - 2\bar{\theta}\left(\hat{\theta} + \underline{\theta}\right) + \left(\hat{\theta} + \underline{\theta}\right)^2\right]}{2\left(2c_H - c_L\right)\left(\bar{\theta} - \underline{\theta}\right)\left(2\bar{\theta} - \underline{\theta} - \hat{\theta}\right)} \tag{66}$$

When the monopolists chooses maximum differentiation, from (15), demand for the pirated good is

$$x_{2P} = \frac{2c_H\left(\bar{\theta} + \hat{\theta} - \underline{\theta}\right) - c_L\left(3\bar{\theta} - 2\underline{\theta}\right)}{2\left(2c_H - c_L\right)} \tag{67}$$

which is positive when $c_L < c_H\frac{2\left(\bar{\theta}+\hat{\theta}-\underline{\theta}\right)}{3\bar{\theta}-2\underline{\theta}}$. ■

Proof of Proposition 8 $\Pi_{\max}$ is an increasing and convex function in c_L since prices and demand functions for both goods are always non-decreasing in c_L.The second derivative of $\Pi_{\max}$ is

$$\frac{d^2\Pi_{\max}}{dc_L^2} = \frac{2c_H^2\left(2\bar{\theta} - \hat{\theta} - \underline{\theta}\right)}{\left(\bar{\theta} + \underline{\theta}\right)\left(2c_H - c_L\right)^3} \tag{68}$$

which is always positive given our assumptions on the parameters.

$\Pi_{\min}$ is always increasing and linear in c_L, as it is immediate to check from (57) and $\Pi_{\min}$ cuts $\Pi_{\max}$ twice for all values of the parameters. Solving $\Pi_{\min} = \Pi_{\max}$, solutions are $\tilde{c}_{L1} = 2c_H\frac{2\bar{\theta}\left(\bar{\theta}-\hat{\theta}-\underline{\theta}\right)+\left(\hat{\theta}+\underline{\theta}\right)^2}{4\bar{\theta}^2-5\bar{\theta}\underline{\theta}-\bar{\theta}\hat{\theta}+2\underline{\theta}^2+2\hat{\theta}\underline{\theta}}$ and $\tilde{c}_{L2} = 2c_H\frac{\bar{\theta}+\hat{\theta}-\underline{\theta}}{3\bar{\theta}-2\underline{\theta}}$. It is immediate to check that $\tilde{c}_{L2} = \bar{c}_L$, the threshold value for c_L that guarantees that piracy is non-negative. Moreover, $\tilde{c}_{L1} > \underline{c}_L$ always.

Finally, is $\tilde{c}_{L1} > \bar{c}_L$? To check this, see whether $2c_H\frac{2\bar{\theta}\left(\bar{\theta}-\hat{\theta}-\underline{\theta}\right)+\left(\hat{\theta}+\underline{\theta}\right)^2}{4\bar{\theta}^2-5\bar{\theta}\underline{\theta}-\bar{\theta}\hat{\theta}+2\underline{\theta}^2+2\hat{\theta}\underline{\theta}} - 2c_H\frac{\bar{\theta}+\hat{\theta}-\underline{\theta}}{3\bar{\theta}-2\underline{\theta}} > 0$, that is $\frac{2\bar{\theta}^3-\bar{\theta}^2\underline{\theta}-4\underline{\theta}^2\hat{\theta}-9\bar{\theta}^2\hat{\theta}+4\bar{\theta}\hat{\theta}^2-4\hat{\theta}^2\underline{\theta}+12\bar{\theta}\underline{\theta}\hat{\theta}}{\left(3\bar{\theta}-2\underline{\theta}\right)\left(4\bar{\theta}^2-5\bar{\theta}\underline{\theta}-\bar{\theta}\hat{\theta}+2\underline{\theta}^2+2\hat{\theta}\underline{\theta}\right)} > 0$. The denominator of this expression is always positive. In fact, $4\bar{\theta}^2 - 5\bar{\theta}\underline{\theta} - \bar{\theta}\hat{\theta} + 2\underline{\theta}^2 + 2\hat{\theta}\underline{\theta} < 0$ iff $\hat{\theta} < \frac{4\bar{\theta}^2-5\bar{\theta}\underline{\theta}+2\underline{\theta}^2}{\bar{\theta}-2\underline{\theta}}$ which is always true as $2\bar{\theta} > \hat{\theta}$.We must now check the numerator $2\bar{\theta}^3 - \bar{\theta}^2\underline{\theta} - 4\underline{\theta}^2\hat{\theta} - 9\bar{\theta}^2\hat{\theta} + 4\bar{\theta}\hat{\theta}^2 - 4\hat{\theta}^2\underline{\theta} + 12\bar{\theta}\underline{\theta}\hat{\theta} > 0$, which happens iff $\hat{\theta} > 2\bar{\theta} - \underline{\theta}$ and $\hat{\theta} < \frac{\bar{\theta}^2}{4\left(\bar{\theta}-\underline{\theta}\right)}$. Notice however that $\hat{\theta} < 2\bar{\theta} - \underline{\theta}$ always. Hence $\tilde{c}_{L1} > \bar{c}_L$ iff $\hat{\theta} < \frac{\bar{\theta}^2}{4\left(\bar{\theta}-\underline{\theta}\right)}$, that is the value $\hat{\theta}_1$ defined in the proposition.

To show part 1. of the proposition is suffices to recall that it must be $c_L < \bar{c}_L$ in order for piracy to be non-negative and that $\Pi_{\max} > \Pi_{\min}$ for all $c_L < \bar{c}_L$. We can therefore conclude $\Pi_{\max} > \Pi_{\min}$ over the whole relevant range for c_L. Figure 1 illustrates this situation.

To show part 2. of the proposition, consider that $\hat{\theta} > \hat{\theta}_1$ implies $\tilde{c}_{L1} < \bar{c}_L$, so that, for all $\underline{c}_L < c_L < \tilde{c}_{L1}$, $\Pi_{\max} > \Pi_{\min}$ and for all $\tilde{c}_{L1} < c_L < \bar{c}_L$, $\Pi_{\max} < \Pi_{\min}$.This second case is illustrated by Figure 3. ■

Proof of Proposition 9 If $c_L > c_H \frac{\bar\theta-\hat\theta}{\bar\theta-\underline\theta}$, then $\Pi^* > \Pi^{**}$(see Proposition 4). Then $\Pi^{diff} > \Pi^*$ iff

$$\frac{c_L\left(\hat\theta-\underline\theta\right)^2 + c_H\left(\bar\theta-\hat\theta\right)\left(2\hat\theta-\underline\theta\right)}{\left(\bar\theta-\underline\theta\right)\left(2\hat\theta-\underline\theta\right)} > c_L \tag{69}$$

that is iff

$$c_L\left(\hat\theta^2-\bar\theta\left(2\hat\theta-\underline\theta\right)\right) + c_H\left(\bar\theta-\hat\theta\right)\left(2\hat\theta-\underline\theta\right) > 0 \tag{70}$$

Condition(70) is always true if $\bar\theta < \frac{\hat\theta^2}{2\hat\theta-\underline\theta}$. If instead $\bar\theta > \frac{\hat\theta^2}{2\hat\theta-\underline\theta}$, it is satisfied if and only if

$$c_L < c_H \frac{\left(\bar\theta-\hat\theta\right)\left(2\hat\theta-\underline\theta\right)}{\bar\theta\left(2\hat\theta-\underline\theta\right)-\hat\theta^2} \tag{71}$$

Notice that condition (71) is compatible with (11), because $\frac{\bar\theta-\hat\theta}{\bar\theta-\underline\theta} < \frac{\left(\bar\theta-\hat\theta\right)\left(2\hat\theta-\underline\theta\right)}{\bar\theta\left(2\hat\theta-\underline\theta\right)-\hat\theta^2}$.
If instead $c_L < c_H \frac{\bar\theta-\hat\theta}{\bar\theta-\underline\theta}$, then $\Pi^{**} > \Pi^*$(see Proposition 4 again). Then $\Pi^{diff} > \Pi^{**}$ iff

$$\frac{c_L\left(\hat\theta-\underline\theta\right)^2 + c_H\left(\bar\theta-\hat\theta\right)\left(2\hat\theta-\underline\theta\right)}{\left(\bar\theta-\underline\theta\right)\left(2\hat\theta-\underline\theta\right)} > c_H \frac{\bar\theta-\hat\theta}{\bar\theta-\underline\theta} \tag{72}$$

which is always true. It then follows that under either equilibrium strategy piracy is eliminated. ■

Proof of Proposition 10 Comparing $\Pi_{\min}$ and Π^* (given respectively by expressions (57) and (7)) we find that $\Pi_{\min} > \Pi^*$ when $\hat\theta < \frac{\bar\theta^2}{4\left(\bar\theta-\underline\theta\right)} = \hat\theta_1$. The strategy of producing one product and selling it to the whole market is never adopted in equilibrium. This implies that if $\Pi_{\max} > \Pi_{\min}$ for the relevant range of c_L (which happens if $\hat\theta < \hat\theta_1$), then $\Pi_{\min} > \Pi^*$ (see proposition 8).

We now prove that $\Pi_{\max}$ crosses Π^{**} (profit from producing one good allowing piracy, given by expression (10)) once.

Comparing $\Pi_{\max}$ and Π^{**} we see that $\Pi_{\max} < \Pi^{**}$ for all $R_1 < c_L < R_2$, where

$$R_1 = \frac{c_H}{\bar\theta^2}\left[-\left(2\bar\theta^2-4\bar\theta\hat\theta+\hat\theta^2\right)+\underline\theta\left(2\bar\theta-\hat\theta\right)-\left(2\bar\theta-\hat\theta-\underline\theta\right)\sqrt{2\bar\theta^2-4\bar\theta\hat\theta+\hat\theta^2}\right],$$

and

$$R_2 = \frac{c_H}{\bar{\theta}^2}\left[-\left(2\bar{\theta}^2 - 4\bar{\theta}\hat{\theta} + \hat{\theta}^2\right) + \underline{\theta}\left(2\bar{\theta} - \hat{\theta}\right) + \left(2\bar{\theta} - \hat{\theta} - \underline{\theta}\right)\sqrt{2\bar{\theta}^2 - 4\bar{\theta}\hat{\theta} + \hat{\theta}^2}\right]$$

It is possible to prove that $R_1 < 0$. To check this see that $R_1 < 0$ for $\bar{\theta} > \hat{\theta} + \frac{1}{2}\sqrt{2\hat{\theta}^2 + 2\underline{\theta}^2}$ and $\bar{\theta} < \hat{\theta} - \frac{1}{2}\sqrt{2\hat{\theta}^2 + 2\underline{\theta}^2}$ but, as $\bar{\theta} > 2\hat{\theta}$, it must be $\bar{\theta} > \hat{\theta} + \frac{1}{2}\sqrt{2\hat{\theta}^2 + 2\underline{\theta}^2}$ always. Define $c_L^{pir} = R_2$.

Then we show that $\Pi_{\max}$ crosses Π^{**} once for $c_L > \underline{c}_L$ and, for $c_L < c_L^{pir}$, $\Pi^{**} > \Pi_{\max}$, whereas $\Pi^{**} < \Pi_{\max}$ for $c_L > c_L^{pir}$. To see this just check that, for $c_L = \underline{c}_L$, $\Pi_{\max} < \Pi^{**}$. In fact,

$$\Pi^{**} - \Pi_{\max}\big|_{c_L = \underline{c}_L} = \frac{c_H \bar{\theta}\left(\bar{\theta} - 2\hat{\theta}\right)}{2\left(\bar{\theta} - \underline{\theta}\right)\left(3\bar{\theta} - 2\hat{\theta}\right)} > 0$$

(recall that $\bar{\theta} > 2\hat{\theta}$ and Π^{**} is invariant with respect to c_L). We have shown in the proof to proposition 8 that $\Pi_{\max}$ is increasing and convex, so $\Pi_{\max}$ cuts Π^{**} from below and is smaller than Π^{**} for $c_L < c_L^{pir}$ and larger for $c_L > c_L^{pir}$. However, while $c_L^{pir} > \underline{c}_L$ always, $c_L^{pir} < \bar{c}_L$ iff $\hat{\theta} > \hat{\theta}_2$, where $\hat{\theta}_2 = \frac{\bar{\theta}^2 - 4\bar{\theta}\underline{\theta} - 4\underline{\theta}^2}{2\left(3\bar{\theta} - 2\underline{\theta}\right)}$. When $c_L^{pir} > \bar{c}_L$, $\Pi^{**} > \Pi_{\max}$ always in the relevant range of c_L.

Recall that in part 1. of the proposition $\hat{\theta} < \hat{\theta}_1$. Therefore $c_L^{pir} < \bar{c}_L$ iff $\hat{\theta}_2 < \hat{\theta} < \hat{\theta}_1$. To have $\hat{\theta}_2 < \hat{\theta}_1$ it must be that $\bar{\theta} > \left(3 + \sqrt{5}\right)\underline{\theta}$. When this happens, we can have two cases. 1) $\hat{\theta}_2 < \hat{\theta} < \hat{\theta}_1$, that is $c_L^{pir} < \bar{c}_L$. In this case, we have shown that $\Pi_{\max} > \Pi_{\min}$ when $\hat{\theta} < \hat{\theta}_1$. We can then conclude that, for all $c_L < c_L^{pir}$, $\Pi^{**} > \Pi_{\max}$ and for all $c_L > c_L^{pir}$, $\Pi_{\max} > \Pi^{**}$. Figure 4 illustrates this situation. 2) $\hat{\theta} < \hat{\theta}_2 < \hat{\theta}_1$ and $c_L^{pir} > \bar{c}_L$. In this case, the largest profit is Π^{**} in the whole relevant range.

Conversely, when $\bar{\theta} < \left(3 + \sqrt{5}\right)\underline{\theta}$, $c_L^{pir} > \bar{c}_L$ and $\Pi_{\max} < \Pi^{**}$ in the whole relevant range.

To prove part 2., recall that when $\hat{\theta} > \hat{\theta}_1$, $\Pi^{*} > \Pi_{\min}$ and $\tilde{c}_{L1} < \bar{c}_L$. Hence, $\bar{c}_L$ is the largest solution to $\Pi_{\max} = \Pi_{\min}$.

$\Pi_{\max}$ crosses $\Pi^{*} = c_L$ twice. Solve $\Pi_{\max} = c_L$ and obtain two solutions

$$\tilde{c}_{L3} = \frac{c_H\left[\left(4\bar{\theta}^2 - \bar{\theta}\hat{\theta} - 5\bar{\theta}\underline{\theta} + 2\underline{\theta}\left(\underline{\theta} + \hat{\theta}\right)\right) - \sqrt{\left(2\bar{\theta} - \hat{\theta} - \underline{\theta}\right)^2\left(4\hat{\theta}\left(\bar{\theta} - \underline{\theta}\right) - \bar{\theta}^2\right)}\right]}{\left(5\bar{\theta}^2 - 2\bar{\theta}\left(\hat{\theta} + 3\underline{\theta}\right) + 2\underline{\theta}\left(\hat{\theta} + \underline{\theta}\right)\right)}$$

$$\tilde{c}_{L4} = \frac{c_H\left[\left(4\bar{\theta}^2 - \bar{\theta}\hat{\theta} - 5\bar{\theta}\underline{\theta} + 2\underline{\theta}\left(\underline{\theta} + \hat{\theta}\right)\right) + \sqrt{\left(2\bar{\theta} - \hat{\theta} - \underline{\theta}\right)^2\left(4\hat{\theta}\left(\bar{\theta} - \underline{\theta}\right) - \bar{\theta}^2\right)}\right]}{\left(5\bar{\theta}^2 - 2\bar{\theta}\left(\hat{\theta} + 3\underline{\theta}\right) + 2\underline{\theta}\left(\hat{\theta} + \underline{\theta}\right)\right)}$$

It can be readily checked that $\tilde{c}_{L3} < \bar{c}_L < \tilde{c}_{L4}$, Moreover, $\tilde{c}_{L3} < \underline{c}_L$. To prove this, consider that, if $\underline{c}_L < \tilde{c}_{L3}$, then, at $c_L = \underline{c}_L$, it must be

$\Pi_{\max} > \Pi^*$. This is verified for all $\hat{\theta} < \hat{\theta}^{'} = \frac{1}{2}\left[\left(\bar{\theta} + \underline{\theta}\right) - \sqrt{\underline{\theta}}\sqrt{2\bar{\theta} - 3\underline{\theta}}\right]$ and $\hat{\theta} > \hat{\theta}^{''} = \frac{1}{2}\left[\left(\bar{\theta} + \underline{\theta}\right) + \sqrt{\underline{\theta}}\sqrt{2\bar{\theta} - 3\underline{\theta}}\right]$. Considering that we are in the case where $\hat{\theta} < \frac{\bar{\theta}}{2}$ (see Lemma 5) and that $\hat{\theta}^{'} < \frac{\bar{\theta}}{2} < \hat{\theta}^{''}$ the relevant value is $\hat{\theta}^{'}$. Check that $\hat{\theta}_1 > \hat{\theta}^{'}$ always, so the only feasible case is $\hat{\theta}^{'} < \hat{\theta} < \frac{\bar{\theta}}{2}$ (the other possible case, $\hat{\theta} < \hat{\theta}^{'}$ is ruled out by the fact that $\hat{\theta}_1 > \hat{\theta}^{'}$ and we are considering $\hat{\theta} > \hat{\theta}_1$). Then, at $c_L = \underline{c}_L$, $\Pi_{\max} < \Pi^*$ always and $\tilde{c}_{L3} < \underline{c}_L$, Therefore in the relevant range for c_L ($\underline{c}_L < c_L < \bar{c}_L$) $\Pi^* > \Pi_{\min}$ and $\Pi^* > \Pi_{\max}$.

Since differentiation is never an optimal strategy in this case, the monopolist will produce one quality either by setting such a low price that piracy is eliminated or by setting a higher price and allowing some piracy. The latter strategy, which yields profits Π^{**}, is more profitable than the former (whose corresponding profits are Π^*) if and only if $c_L < c_H \frac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}}$ (see Proposition 4). This case is depicted in Figure 5.

Notice that $c_H \frac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}} > \underline{c}_L$, therefore it is never the case that $\Pi^* > \Pi^{**}$ on the whole range $(\underline{c}_L, \bar{c}_L)$, whereas Π^{**} is overall greater than Π^* in the range $(\underline{c}_L, \bar{c}_L)$ if $c_H \frac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}} > \bar{c}_L$, that is if $\hat{\theta} > \frac{\bar{\theta}^2 + 2\bar{\theta}\underline{\theta} - 2\underline{\theta}^2}{5\bar{\theta} - 4\underline{\theta}}$. ∎

References

[1] Besen, S., and S. Kirby (1989), Private Copying, Appropriability, and Optimal Copying Royalties, *Journal of Law and Economics*, 32, 255-280.

[2] Conner, K., and R. Rumelt (1991), Software Piracy: An Analysis of Protection Strategies, *Management Science*, 37, 125-139.

[3] Crampes, C., and A. Hollander (1995), Duopoly and Quality Standards, *European Economic Review*, 39, 71-82.

[4] Gabszewicz, J.J., A. Shaked, J. Sutton, and J.-F. Thisse (1986), Segmenting the Market: The Monopolist Optimal Product Mix, *Journal of Economic Theory*, 39(2), 273-289.

[5] Gayer, A., and O. Shy (2001), Internet, Peer-to-Peer, and Intellectual Property in Markets for Digital Products, *mimeo*, University of Haifa.

[6] Hui, K.L., Png, I.P.L., and Y. Cui (2001), Piracy and the Demand for Legitimate Recorded Music, *mimeo*, National University of Singapore, downoadable from http://www.ssrn.com/.

[7] Itoh, M. (1983), Monopoly, Product Differentiation and Economic Welfare, *Journal of Economic Theory*, 31, 88-104.

[8] Johnson, W. (1985), The Economics of Copying, *Journal of Political Economy*, 93, 158-174.

[9] Kahin, B., and H. Varian (eds.) (2000), *Internet Publishing and Beyond: The Economics of Digital Information and Intellectual Property.* Cambridge, MA: The MIT Press.

[10] Lambertini, L. (1997), The Multiproduct Monopolist under Vertical Differentiation: An Inductive Approach, *Recherches Economiques de Louvain*, 63(2), 109-22.

[11] Liebowitz, S. (1985), Copying and Indirect Appropriability: Photocopying of Journals, *Journal of Political Economy*, 93, 945-957.

[12] Maskin E., and J. Riley (1984), Monopoly with Incomplete Information, *Rand Journal of Economics*, 15(2), 171-196.

[13] Motta, M. (1993), Endogenous Quality Choice: Price vs. Quantity Competition, *Journal of Industrial Economics*, 41(2), 113-31.

[14] Mussa, E., and S. Rosen (1978), Monopoly and Product Quality, *Journal of Economic Theory*, 18, 171-196.

[15] Spence, M. (1975), Monopoly, Quality and Regulation, *Bell Journal of Economics*, 6, 417-429.

[16] Shaked, A., and J. Sutton (1982), Relaxing Price Competition through Product Differentiation, *Review of Economic Studies*, 49, 3-13.

[17] Shy, O. (2001), *The Economics of Network Industries*, Cambridge University Press.

[18] Shy, O., and J.-F. Thisse (1999), A Strategic Approach to Software Protection, *Journal of Economics and Management Strategy*, 8, 163-190.

[19] Tirole, J. (1988), *The Theory of Industrial Organization*, Cambridge, MA: MIT University Press.

[20] Varian, Hal R. (2000), Buying, Sharing and Renting Information Goods, *Journal of Industrial Economics*, 48(4), 473-88.