Efficient Distribution of Copyright Revenue

Abstract

Habitually, copyright royalties are set at a specific percentage of sales. In this paper, we consider the efficiency of such an arrangement. In particular, we show that in the most realistic scenario in which the final sales of the creation that generates copyright income is subject to risk, then the only case in which we can guarantee that the efficient contract will involve a fixed percentage royalty is when both parties to the contract (creator and distributor) have constant and common relative risk aversion. Out of all efficient contracts, that which is finally selected will depend on some type of bargaining process. For the case of constant and common risk aversion, in the Nash bargaining model copyright revenue should be shared in proportions equal to the relative bargaining powers of the creator and the distributor.

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1 Introduction

A very common aspect of the production and distribution of cultural creations is the fact that the creator does not also participate directly in the distribution or sale of the creation (more correctly, the distribution or sale of the rights in the creation). The reasons for this are, quite obviously, based on the economic theory of specialization on the one hand, but also for the case of the reproduction rights in easily reproducible creations (for example, musical compositions and written works), the transactions costs savings that accrue from having a copyright collective that deals with many similar creations is perhaps the single most significant reason for creators not dealing directly with final consumers. Naturally, when the creator does not take care of the entire production-distribution chain, the matter of how she[1] is reimbursed from the income that is generated becomes important. We shall refer to the creator's reimbursement as the "copyright royalty".

Clearly, there are a great many alternatives from which the formula that determines royalty payments may be selected. However, in spite of the size of the alternative set, one particular format is very much prevalent in real-world situations - the royalty is a fixed proportion of total sales revenue (see, for example, Baumol and Heim (1967), and Towse (2001), where a wealth of real-world information is given concerning royalty contracts). Assuming that each consumption unit is sold at a constant price, naturally this is equivalent to the creator receiving a fixed proportion of the sales price of each unit sold. We shall refer to this as a fixed proportion sharing rule, since independently of the amount of sales revenue that ends up being collected, the royalty percentage payment is the same. In contrast, for example, the author may be offered a lower percentage share if the sales revenue is small, and a higher percentage share for greater sales revenue outcomes. The principle objective of this paper is to investigate whether or not such a sharing rule is indeed efficient.

The issue of efficiency will be studied quite apart from any issue regarding incentives. For example, any sharing rule that states that the creator's royalty will be calculated as an increasing

[1] Throughout, we shall use the feminine pronoun to refer to a creator, and the masculine to refer to a distributor. This is purely in the interests of ease and clarity of exposition, and no futher connotations should be attached to this decision.

function of sales income means that the creator is interested in maximizing sales income, while the distributor is interested in maximizing profits. Aside from the special case of constant elasticity of demand, the two points will not (in general) coincide. Hence the pricing decision of the distributor will not, in general, coincide with the pricing decision that the creator would most prefer. In this paper, we simply take the total sales revenue as being a random variable, and we do not consider what in fact affects the values that it takes. The only question that we set out to answer is exactly how this revenue should be shared between the creator and the distributor.

The analysis in the paper is sufficiently general to cover any type of distribution agreement. For example, if the creator is a musician, the distributor may be a record label (also often referred to as the publisher), who fixes the creation (in this case, the musical composition) onto some physical support, copies of which are then sold to consumers. The royalty contract will stipulate how much the record company must pay the musician out of final sales revenue. However, for the case of musical compositions, there is also a large market for broadcasting and public communication rights, which is typically managed by a copyright society. Once again, the revenues that the copyright society generates must somehow be shared among the members (the musicians), and the way that this is done is what we are calling a royalty contract here. In general then, although different rights may be marketed and administered by different distributors, the same problem arises when the revenue that is generated must be split between the creator and the distributor, and this is the problem that we study here.

Since we want to decide how the sales revenue should be shared between the distributor and the creator under the assumption that sales revenue can take any of a set of different values, we are necessarily thrust into the realm of sharing a random variable. There is uncertainty as to the final value of sales revenue that will be available to be shared, but the contract that stipulates how the revenue will be shared in each contingency must be decided before the true state of nature has been revealed. As is natural, we shall only consider Pareto efficient sharing rules, that is, rules under which neither party to the contract can benefit from a change without making the other party worse off. Thus, we are considering a direct application of the theory of efficient risk sharing, initiated (in the modern sense) by Borch (1962)[2] . However, the relevant literature has shown to be of little practical aid for real life cases, due in no small part to the very complex nature of the subject matter. Hence, in this paper we shall concentrate on attempting to provide a simple model, with transparent results and conclusions, that we hope will be at least a first step towards characterizing the type of royalty contract that should be observed in real life cases. This paper really contains no new results or theories, but rather our intention is to simplify existing theories and results in order to attempt to make them more useful to real-life situations.

In what follows, we firstly consider a very simple theory of efficient risk sharing, with the intention to find out if indeed we can identify situations in which an efficient royalty contract will always imply a fixed proportion sharing rule, and situations in which this will never be the case. Then, we go on to analyze a particular model, the Nash bargaining model, from which an exact contract can be identified. In this case, we can consider the cases in which a fixed proportion sharing rule is always efficient, with the objective of comparing the exact contract that eventuates with the type of contract that is used in real-life royalty contracts. Section 4 concludes and offers certain directions for further research.

2 Efficient sharing rules

To begin with, we should consider what we mean by a sharing rule, and what we mean by an efficient sharing rule. In general, let us assume that total sales revenue is represented by the variable x. In interests of simplicity, throughout the paper we assume that x is a binomial random variable, that takes the value x_1 with probability $1-p$, and the value x_2 with probability p. Without loss of generality, we assume $x_2 < x_1$, that is, the bad state of the world is state 2. Both creator and distributor are fully agreed on the value of the probability p. We assume that the creator has utility function $u_c(z)$, and that the distributor has utility function $u_d(z)$, where both functions are assumed to be strictly increasing and strictly concave (i.e. both the creator and the distributor are assumed to prefer more money to less, and are strictly risk averse). For simplicity,

[2] See also the seminal papers of Wilson (1968) and Pratt and Zeckhauser (1989). Gollier (1991) provides a summary of the relevant literature up to 1991, and Pratt (2000) shows how certain previous results can be unified into a consistent theory.

we assume that $u_j(0) = 0$ for $j = c, d$.

A sharing rule is a function, $k(x)$, where it is agreed that the creator will receive as royalty income the amount $k(x)$, and that the distributor will receive as his payment the amount $x - k(x)$. Since we have assumed that x is binomial, we can restrict our analysis to only two numbers, $k_1 \equiv k(x_1)$ and $k_2 \equiv k(x_2)$. It is interesting to note that nothing at all is gained by assuming that the contract stipulates an up-front payment and then some sharing arrangement of final revenue, as is often the case (see Baumol and Heim (1967) for a discussion of the types of contract that are often used, at least for the case of books). This is because, whatever is the initial up-front payment, say h_0, and the ensuing royalty payments, $h_1(h_0, x_1)$ and $h_2(h_0, x_2)$, our model is simply set as $k_1 = h_0 + h_1$ and $k_2 = h_0 + h_2$. So long as the up-front option is always set efficiently, it can always be described by the simpler two payment model that we use here.

We assume that if no contract is agreed upon, both parties receive an income of 0. Since we are assuming binomial risk, both parties to the contract will act in accordance with the expected utility theory, that is, they will maximize their respective expected utility functions (see Von Neumann and Morgenstern (1947))

$$Eu_j(z^j) = pu_j(z_1^j) + (1-p)u_j(z_2^j) \text{ for } j = c, d$$

where $z_i^c = k_i,\ z_i^d = x_i - k_i$ for $i = 1, 2$.

Given this, all of our analysis can be set out graphically in the plane of vectors $k = (k_1, k_2)$. In this space, it is immediate that the creator's indifference curves are convex to the origin and geometrically higher curves correspond to greater expected utility, while the distributor's indifference curves are concave to the origin and those corresponding to greater expected utility values are located geometrically lower. The creator's reservation utility is given by the indifference curve that passes through the origin, while the distributor's reservation utility is given by the indifference curve that passes through the point (x_1, x_2). This information is shown in figure 1.

In figure 1 we have also shown the certainty contracts for each of the two participants. The creator will receive a certain payoff whenever $k_2 = k_1$, which is a line of slope 1 that passes through the origin, while the distributor will receive a certain payoff whenever $x_2 - k_2 = x_1 - k_1$, that is,

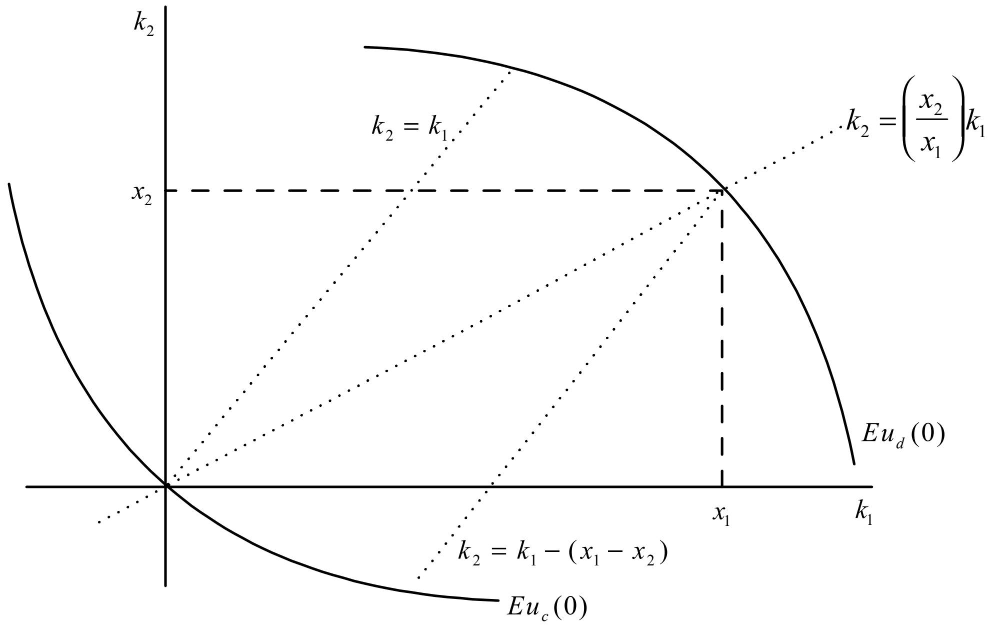

Figure 1:

whenever $k_2 = k_1 - (x_1 - x_2)$, which is a line of slope 1 that passes through the point (x_1, x_2). Given the assumption that $x_1 > x_2$, which is referred to as the existence of aggregate risk, the certainty line of the creator is everywhere above the certainty line of the distributor. Finally, we have also shown the point that correspond to the contracts that specify a fixed proportional share of revenue, the line $k_2 = \left(\frac{x_2}{x_1}\right) k_1$, that passes through the origin and the point (x_1, x_2)[3] .

An efficient contract is defined as one that satisfies Pareto efficiency. A contract $k = (k_1, k_2)$ is Pareto efficient if there does not exist any other contract $\widetilde{k} = (\widetilde{k}_1, \widetilde{k}_2)$ such that $Eu_j(z^j(\widetilde{k})) > Eu_j(z^j(k))$ for $j = 1, 2$ simultaneously. In words, a contract is Pareto efficient if and only if there does not exist another contract that both the creator and the distributor prefer to the first. As was first shown by Borch (1962), any efficient risk sharing mechanism must satisfy the condition that the marginal rates of substitution of each participant are equal. Since the marginal rate of substitution of participant j at a point k is simply

$$MRS_j(k) = -\frac{\left(\frac{\partial Eu_j(z^j)}{\partial k_1}\right)}{\left(\frac{\partial Eu_j(z^j)}{\partial k_1}\right)} = -\frac{(1-p)}{p}\frac{\left(\frac{\partial u_j(z_1^j)}{\partial k_1}\right)}{\left(\frac{\partial u_j(z_2^j)}{\partial k_2}\right)} \text{ for } j = c, d$$

[3] We refer to this second point as the distributor's origin.

we can identify all efficient contracts as those points k that satisfy

$$u'_c(k_1)u'_d(x_2 - k_2) = u'_c(k_2)u'_d(x_1 - k_1) \tag{1}$$

Note that (1) identifies implicitly a function, $k_2 = K(k_1)$, that corresponds to all efficient contracts. This function is normally called the contract curve of the problem. At any point at which the contract curve intersects the line $k_2 = \left(\frac{x_2}{x_1}\right) k_1$, we have an efficient contract that has a fixed proportional sharing rule. Furthermore, if the contract curve coincides with the line $k_2 = \left(\frac{x_2}{x_1}\right) k_1$, then *all* efficient contracts have a fixed proportional sharing rule. This is the particular case that we would like to identify. We now go on to point out some useful results concerning the contract curve[4] .

Lemma 1 *For all* $0 < k_i < x_i$ *for* $i = 1, 2$*, the contract curve satisfies* $k_1 > K(k_1) > k_1 - (x_1 - x_2)$*.*

Proof. If we use $k_2 \geq k_1$, then clearly $u'_c(k_1) \geq u'_c(k_2)$, and so in order for (1) to hold, we require $u'_d(x_2 - k_2) \leq u'_d(x_1 - k_1)$. However, this requires $x_2 - k_2 \geq x_1 - k_1$, that is, $k_1 - k_2 \geq x_1 - x_2$. However, this equation is impossible, since its right-hand-side is non-negative, and its left-hand-side is strictly negative. Similarly, if $k_2 - x_2 \leq k_1 - x_1$, that is, $x_2 - k_2 \geq x_1 - k_1$, then $u'_d(x_2 - k_2) \leq u'_d(x_1 - k_1)$. Thus, we require $u'_c(k_1) \geq u'_c(k_2)$, that is, $k_1 \leq k_2$ which was just shown to be impossible. ■

The intuition for Lemma 1 is very simple - given two strictly risk averse participants, an efficient contract can never ask that only one of them accepts all the risk. Hence the contract cannot satisfy $k_2 = k_1$ (creator suffers no risk) nor can it satisfy $x_2 - k_2 = x_1 - k_1$ (distributor suffers no risk). The particular cases of extreme values of k_1 must be excluded, since these are equivalent (for one of the two participants) to not signing a contract at all. In any case, the next Lemma shows why these extreme cases may be important and should be discussed separately.

Lemma 2 *If the utility functions satisfy* $u'_j(0) = \infty$ *for* $j = c, d$*, and assuming that the values of* k_i *are finite, then the contract curve passes through the origin and the point* (x_1, x_2)*.*

Proof. Put $k_1 = 0$ into equation (1). The right hand side must now take the value ∞, and so in order for it to be satisfied, we require the left hand side to also take the value ∞. This can

[4] Some of the following results are also given in Watt (2000) for a very similar setting.

only occur if $k_2 = 0$. Hence the contract curve passes through the origin. Secondly, put $k_1 = x_1$ into (1) and note that by a similar argument, the equation can only be satisfied if $k_2 = x_2$. ∎

Lemma 3 *The slope of the contract curve at any point is*

$$K'(k_1) = \frac{A_c(k_1) + A_d(x_1 - k_1)}{A_c(k_2) + A_d(x_2 - k_2)}$$

where $A_j(z) = -\frac{u_j''(z)}{u_j''(z)}$ *is the Arrow-Pratt measure of absolute risk aversion of party* j.

Proof. First, take logarithms of each side of (1) so that it reads

$$Ln(u_c'(k_1)) + Ln(u_d'(x_2 - k_2)) = Ln(u_c'(k_2)) + Ln(u_d'(x_1 - k_1))$$

that is

$$h(k_1, k_2) \equiv Ln(u_c'(k_1)) + Ln(u_d'(x_2 - k_2)) - [Ln(u_c'(k_2)) + Ln(u_d'(x_1 - k_1))] = 0$$

Now, from the implicit function theorem, and recalling that by definition

$$\frac{\partial Ln(u'(z))}{\partial z} = -A(z)$$

we have

$$\begin{aligned} \frac{dk_2}{dk_1} &= -\frac{\left(\frac{\partial h(k)}{\partial k_1}\right)}{\left(\frac{\partial h(k)}{\partial k_2}\right)} \\ &= \frac{A_c(k_1) + A_d(x_1 - k_1)}{A_c(k_2) + A_d(x_2 - k_2)} \end{aligned}$$

as required. ∎

Corollary 1 $K'(k_1) > 0$.

Proof. This follows immediately from Lemma 3 when both parties are strictly risk averse as has been assumed. ∎

Corollary 2 *If both parties to the contract have decreasing (increasing, constant) absolute risk aversion, then* $K'(k_1) < (>, =)1$.

Proof. If both parties to the contract have decreasing absolute risk aversion, then $A_j(z_1) < A_j(z_2)$ whenever $z_1 > z_2$. However, under Lemma 1, all points on the contract curve will satisfy $k_1 > k_2$ and $x_1 - k_1 > x_2 - k_2$. Hence, if both parties to the contract have decreasing absolute risk aversion, the numerator of $K'(k_1)$ in Lemma 3 must be smaller than the denominator, that is, $K'(k_1) < 1$. Similar arguments suffice to prove the other two cases. ∎

We can now state our first main result:

Theorem 1 *The contract curve has at least one point satisfying* $0 \leq k_1 \leq x_1$*, such that the contract stipulates a fixed proportional sharing rule. Furthermore, if* $u_j'(0) < \infty$ *for both* $j = c, d$*, and if the two utility functions satisfy* $P_c(z^c) \geq 0$ *and* $2A_d(z^d) \geq P_d(z^d)$*, where* $P(z) \equiv -\frac{u'''(z)}{u''(z)}$ *is the degree of absolute prudence, then there is a unique intersection between the contract curve and the fixes proportion sharing rule line.*

Proof. Firstly, for the case in which $u_j'(0) = \infty$ for at least one $j = c, d$, the result is a direct application of Lemma 2. In this case, the contract curve touches the fixed proportion sharing rule line at the origin of the individual for whom $u_j'(0) = \infty$, which of course, may be for both individuals (i.e. in this case, there may be two points at which the contract curve touches the fixed proportion sharing rule line).

For the other case, $u_j'(0) < \infty$ for both $j = c, d$, we proceed as follows. Firstly, recall that all possible fixed proportion sharing rules are defined by the equation $k_2 = \delta k_1$, where $\delta \equiv \frac{x_2}{x_1}$. If the contract curve touches the fixed proportion sharing rule line, then the two marginal rates of substitution must be equal, that is we must have

$$\frac{u_c'(k_1)}{u_c'(\delta k_1)} = \frac{u_d'(x_1 - k_1)}{u_c'(\delta(x_1 - k_1))} \implies \frac{u_c'(k_1)}{u_d'(x_1 - k_1)} = \frac{u_c'(\delta k_1)}{u_c'(\delta(x_1 - k_1))}$$

Now, define two functions as follows

$$M_1(k_1) \equiv \frac{u_c'(k_1)}{u_d'(x_1 - k_1)} \quad \text{and} \quad M_2(k_1) \equiv \frac{u_c'(\delta k_1)}{u_c'(\delta(x_1 - k_1)}$$

Now, since $M_1(0) = \frac{u_c'(0)}{u_d'(x_1)} > \frac{u_c'(0)}{u_d'(x_2)} = M_2(0)$ and similarly $M_1(x_1) = \frac{u_c'(x_1)}{u_d'(0)} < \frac{u_c'(x_2)}{u_d'(0)} = M_2(x_1)$, and since both are continuous functions, there must be at least one point at which $M_1(k_1) = M_2(k_2)$, and that point must satisfy $0 < k_1 < x_1$. Furthermore,

$$\begin{aligned} M_1'(k_1) &= \frac{u_c''(k_1)u_d'(x_1 - k_1) + u_c'(k_1)u_d''(x_1 - k_1)}{u_d'(x_1 - k_1)^2} \\ &= -M_1(k_1)\left[A_c(k_1) + A_d(x_1 - k_1)\right] \end{aligned}$$

and

$$\begin{aligned} M_2'(k_1) &= \frac{\delta\left[u_c''(\delta k_1)u_d'(\delta(x_1 - k_1)) + u_c'(\delta k_1)u_d''(\delta(x_1 - k_1))\right]}{u_d'(\delta(x_1 - k_1))^2} \\ &= -M_2(k_1)\delta\left[A_c(\delta k_1) + A_d(\delta(x_1 - k_1)\right] \end{aligned}$$

Now, with a minimal amount of effort, the second derivatives of each function can be calculated as

$$M_1''(k_1) = M_1(k_1)\left[(A_c(k_1)+A_d(x_1-k_1))^2 - A_c'(k_1) + A_d'(x_1-k_1)\right] \tag{2}$$

$$M_2''(k_1) = M_2(k_1)\delta^2\left[(A_c(\delta k_1)+A_d(\delta(x_1-k_1)))^2 - A_c'(\delta k_1) + A_d'(\delta(x_1-k_1))\right] \tag{3}$$

However, since

$$A'(z) = -\left(\frac{u'''(z)u'(z) - u''(z)u''(z)}{u'(z)^2}\right) = -\frac{u'''(z)}{u'(z)} + A(z)^2 = -P(z)A(z) + A(z)^2$$

the square bracket terms in (2) and (3) can be written as

$$\left[2A_d(x_1-k_1)^2 + 2A_c(k_1)A_d(x_1-k_1) + P_c(k_1)A_c(k_1) - P_d(x_1-k_1)A_d(x_1-k_1)\right]$$

and

$$\left[2A_d(\delta(x_1-k_1))^2 + 2A_c(\delta k_1)A_d(\delta(x_1-k_1)) + P_c(\delta k_1)A_c(\delta k_1) - P_d(\delta(x_1-k_1))A_d(\delta(x_1-k_1))\right]$$

respectively.

Finally, the conditions $P_c(z^c) \geq 0$ and $2A_d(z^d) \geq P_d(z^d)$ guarantee that these two square bracket terms are strictly positive, and so as a result we get $M_i''(k_1) > 0$ for $i = 1, 2$, that is, the functions $M_i(k_1)$ are strictly convex, and so can only have a single intersection. ■

Perhaps the most interesting of these results is Theorem 1, which guarantees that for all cases in which the utility functions satisfy the requisite that the creator has non-negative prudence, and the distributor's prudence is not greater than twice his absolute risk aversion, and $u_j'(0) < \infty$ for both $j = c, d$, exactly one efficient contract exists that stipulates a fixed sharing rule, and that this contract cannot be located at either extreme case (i.e. the contract must share the surplus between the creator and the distributor with a strictly positive share for each). Utility functions that satisfy these properties are, for example, constant absolute risk aversion (in which case $P = A$) and quadratic forms (for which $P = 0$), both of which have been extensively used to study empirical cases, but neither of which satisfy the logical assumption that absolute risk aversion is decreasing. The most usual case of utility functions that do satisfy decreasing absolute risk aversion are those that satisfy constant relative risk aversion (CRRA), which coincide with the case $u_j'(0) = \infty$, and hence have contract curves that pass through the two origins. Note that, from Theorem 1, if we consider a case with $u_j'(0) = \infty$ for both the creator and the distributor, since the contract curve passes through the fixed proportion sharing rule line at two points (the two origins) then it does not have a single intersection, and so cannot satisfy the conditions $P_c(z^c) \geq 0$ and $2A_d(z^d) \geq P_d(z^d)$. Hence in the case of CRRA functions, either there are no internal intersections between the contract curve and the fixed proportion sharing rule line, or the two coincide completely.

The question is whether or not the efficient sharing rule that involves fixed proportions will be the final solution to any bargaining process designed to select a particular contract from the efficient set. However, if the contract curve were to coincide entirely with the line that indicates fixed proportion sharing rules, then independently of exactly which contract is used, it must share the sales revenue in the same proportion in all states, as is commonly done in the real world.

There exist several empirical studies that estimate risk aversion, with a rather common result being that the degree of relative risk aversion at something that is very nearly constant and valued between 0.5 and 2.5 (see, for example, Friend and Blume (1975), Hansen and Singleton (1982), Szpiro (1986), Weber (1970))[5] . Hence, it seems that constant relative risk aversion is a good assumption to place upon our model. In the light of this, we can state the following theorem:

Theorem 2 *If both parties to the contract have constant relative risk aversion with the same degree of relative risk aversion, then all efficient contracts stipulate a royalty contract with a fixed proportion sharing rule.*

Proof. If both the creator and the distributor have utility functions that are consistent with constant relative risk aversion, then we have

$$u_j(z) = \frac{z^{1-R_j}}{(1-R_j)} + y_j \quad \text{for} \quad j = c, d$$

where the y_j are constants. For this function, the Arrow-Pratt measure of relative risk aversion is simply $zA_j(z) = R_j$. Marginal utility is given by

$$u_j'(z) = z^{-R_j} \quad \text{for} \quad j = c, d$$

[5] More recently, Levy (1994) and Blake (1996), using observed portfolio choices, have found much higher values. These results, however, as well as the findings of Mehra and Prescott (1985), imply that relative risk aversion may only be as high as 47.

and so in this case the contract curve, equation (1) which is where the two marginal rates of substitution are equated, can be written as

$$\left(\frac{k_1}{k_2}\right)^{-R_c} = \left(\frac{x_1 - k_1}{x_2 - k_2}\right)^{-R_d} \tag{4}$$

Now, a point that has a fixed proportion sharing rule has $k_2 = \left(\frac{x_2}{x_1}\right) k_1$. At any such point we have

$$\left(\frac{k_1}{k_2}\right)^{-R_c} = \left(\frac{x_1}{x_2}\right)^{-R_c}$$

On the other hand, at any contract that has a fixed proportion sharing rule

$$\begin{aligned}\left(\frac{x_1 - k_1}{x_2 - \left(\frac{x_2}{x_1}\right) k_1}\right)^{-R_d} &= \left(\frac{\left(\frac{x_1 - k_1}{x_2}\right) x_2}{\left(\frac{x_2 x_1 - x_2 k_1}{x_1}\right)}\right)^{-R_d} \\ &= \left(\left(\frac{x_1}{x_2}\right)\left(\frac{x_1 x_2 - k_1 x_2}{x_2 x_1 - x_2 k_2}\right)\right)^{-R_d} \\ &= \left(\frac{x_1}{x_2}\right)^{-R_d}\end{aligned}$$

Hence, a point with a fixed proportion sharing rule only satisfies the contract curve equation (4) if

$$\left(\frac{x_1}{x_2}\right)^{-R_c} = \left(\frac{x_1}{x_2}\right)^{-R_d}$$

Since $x_1 \neq x_2$, this equation can only be satisfied if $R_c = R_d$, in which case it is satisfied at every point with a fixed proportion sharing rule. ■

This theorem gives us the case that we are searching for, that is, the case in which all efficient contracts stipulate a fixed proportion sharing rule. We require that both the creator and the distributor have utility functions that satisfy constant relative risk aversion, and the relative risk aversion parameter for both must be the same (i.e. they are equally risk averse in relative lotteries). However, we can also state the following corollary, the proof of which follows from the proof of the theorem:

Corollary 3 *If both the creator and the distributor have constant relative risk averse preferences but with different degrees of relative risk aversion, that is, $R_c \neq R_d$, then no efficient royalty contracts correspond to a fixed proportion sharing rule.*

Thus, the result that all efficient sharing rules will always correspond to a fixed proportion share of the final revenue is quite delicate, since if the creator and the distributor have constant absolute risk aversion but different values of the risk aversion parameter, then a fixed proportion sharing rule is never efficient! Concretely, if we take the case $R_c > R_d > 1$, then we have

$$\left(\frac{x_1}{x_2}\right)^{-R_c} = \left(\frac{x_2}{x_1}\right)^{R_c} < \left(\frac{x_2}{x_1}\right)^{R_d} = \left(\frac{x_1}{x_2}\right)^{-R_d}$$

Thus, working backwards through the proof of the theorem, we find that the indifference curve of the distributor is steeper as it passes through the fixed proportion sharing rule line than the indifference curve of the creator. Therefore, the contract curve must lie strictly above the fixed proportion sharing rule line for all contracts (except, of course, for the two contracts located at the origins). In this case, the royalty contract stipulates that the creator should get a greater share of sales revenues in the bad state (state 2) and a lower share in the good state. The intuition for this is easy to see. Since the creator is more risk averse than the distributor, it is efficient for the latter to insure (partially) the former by offering a more stable income stream. This can be done by increasing the payment in the bad state and reducing it in the good state. Logically, the (realisitically unlikely) case of $R_c < R_d$ leads to the opposite result.

It is interesting to note that certain real world contracts that are quite often used, at least for the case of author royalties for books, involve a point that is located strictly below the fixed proportion sharing rule line. For example, Baumol and Heim (1967), page 34, note that author's royalties are often graduated upwards, so that the royalty percentage grows as the total sales revenue grows. This implies that the royalty percentage in the good state is greater than that in the bad state, that is, a point located below the fixed proportion sharing rule line. If both the creator and the distributor have constant relative risk aversion (which is actually quite likely, as is mentioned in the next paragraph), then such a contract can only be efficient (i.e. located on the contract curve) if the creator is less risk averse than the distributor, something that is very difficult to believe.

Surely such contracts are highly inefficient. To see this, once again, take the case of music distribution. It is quite well known that record companies quite often give their artists an advance payment on royalties[6] , that is, when the contract is signed, an up-front payment is made as an

[6] Very recently, according to publicly released information in the trade press, Mariah Caray signed with EMI

advance on future royalties (so that the artist will only perceive royalties in the future once the up-front payment amount has been reached)[7] . This is equivalent to the distributor insuring the artist, since if the royalties due never turn out to reach the up-front payment amount, then it is the distributor that suffers the loss. In fact, an up-front royalty advance provides the equivalent of a deductible insurance contract, which is well known to be the best possible option for the insured when the premium is proportionally related to the indeminty (see, for example, Arrow (1971) for the first proof of this result). To see that this is true, simply take the maximum possible royalty payment state as being the "no-loss" state. Then, when the royalty is high but not maximum, a small loss has occurred, and a deductible contract will stipulate that no coverage should be paid in this case. However, when the royalty turns out to be very low (a high loss has occurred), a deductible contract will stipulate full coverage above the deductible, thereby implying that the total loss suffered by the insured cannot be any greater than the amount stipulated by the deductible. This is clearly what a royalty advance contract offers. In any case, in order for the distributor to offer insurance to the creator, it will always be necessary that the former be less risk averse than the latter (indeed, almost all insurance models assume that the insurer is risk neutral), which (assuming constant relative risk aversion) can only imply a point above, never below, the fixed percentage royalty line.

3 The Nash bargaining solution

In the previous section, we have considered efficient royalty contracts between a creator and a distributor. However, there are an infinite number of such contracts (assuming perfect surplus divisibility), and so we still need to look at the mechanism under which exactly how one of these contracts is chosen over the others. Here, we take the Nash bargaining solution as being the relevant mechanism (see Nash (1950)).

The Nash bargaining solution (NBS) chooses one particular point on the contract curve (the set of efficient contracts), according to a small set of reasonable axioms that the solution should

with a 22 million euro advance on royalty payments. No information at all concerning the royalty percentages as a function of sales revenue was given, however.

[7] Baumol and Heim (1967) also note that such contracts are common for book publishing.

adhere to. Nash proved that his axioms can only select exactly one contract, and that particular contract appears as the solution to a very simple constrained maximization problem, namely

$$\max_{x} \ (Eu_1(x) - Eu_1(0))^{\alpha}(Eu_2(x) - Eu_2(0))^{1-\alpha} \ \text{ subject to } \ x \in X$$

where X is the set of feasible bargains, and α reflects the relative bargaining power of agent 1. According to our assumptions, we have $Eu_c(0) = Eu_2(0) = 0$, and only two state of nature, and so in our case the NBS is the solution to the problem

$$\max_{k_1,k_2} \ (pu_c(k_1) + (1-p)u_c(k_2))^{\alpha}(pu_d(x_1 - k_1) + (1-p)u_d(x_2 - k_2))^{1-\alpha}$$

where there is no real need to constrain the values of k_i (even though it may seem reasonable for them to be non-negative and not greater than x_i), since in principle there is no reason to assume that negative incomes for one party or the other cannot be optimal.

Taking the two first order conditions and simplifying, we get immediately to the fact that the NBS, here denoted by the vector $k^* = (k_1^*, k_2^*)$, is found by the simultaneous solution to the two following equations

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{Eu_d(k^*)}{Eu_c(k^*)}\right) = \frac{u_d'(x_1 - k_1^*)}{u_c'(k_1^*)} \tag{5}$$

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{Eu_d(k^*)}{Eu_c(k^*)}\right) = \frac{u_d'(x_2 - k_2^*)}{u_c'(k_2^*)} \tag{6}$$

Combining the two first order conditions, we get

$$\frac{u_d'(x_1 - k_1^*)}{u_c'(k_1^*)} = \frac{u_d'(x_2 - k_2^*)}{u_c'(k_2^*)}$$

that is

$$\frac{u_d'(x_1 - k_1^*)}{u_d'(x_2 - k_2^*)} = \frac{u_c'(k_1^*)}{u_c'(k_2^*)}$$

i.e. as expected, the NBS is obviously a particular point that satisfies the contract curve for the problem.

On the other hand, given any vector $k = (k_1, k_2)$, as we have already seen above, a contract with a fixed proportion sharing rule stipulates

$$\frac{k_1}{x_1} = \frac{k_2}{x_2} \quad \Rightarrow \quad k_2 = \left(\frac{x_2}{x_1}\right) k_1 \tag{7}$$

that is, there is a simple linear relationship between the two royalty payments. Thus, we are interested in knowing if the simultaneous solution to (5) and (6) will, in general, satisfy (7).

Since under Lemma 4, we know that in all cases there does exists at least one efficient contract that stipulates a fixed proportion sharing rule, in principle, we cannot ignore the possibility that, while it is not guaranteed under any case except constant and common relative risk aversion, a fixed proportion sharing rule may eventuate as the optimal contract. For example, in the case of $u_j'(0) < \infty$ for both $j = c, d$, this would only occur if the NBS locates exactly that single efficient contract that lies on the fixed proportion sharing rule line. Naturally, this would be nothing short of a miracle, especially when we consider that for the fixed proportion sharing rule to always eventuate under the NBS, then altering the underlying parameter set would shift the NBS to exactly the new unique point satisfying both efficiency and a fixed proportion sharing rule. Since it is clearly extremely doubtful that we will ever find such a case, we shall concentrate here exclusively on the case for which a fixed proportion sharing rule is always found as the NBS, that is, the case of constant and common relative risk aversion, $R_c = R_d = R$. In particular, since we are assuming a specific utility form, we can also locate exact solutions (numerical if we introduce numerical data for the parameters), which will allow us to get an idea as to the exact value of the royalty proportion in the solution.

For this case, since

$$u_j(z) = \frac{z^{1-R_j}}{(1-R_j)} + y_j \quad \text{for} \quad j = c, d$$

and normalizing (without loss of generality) so that $y_j = 0$ for both $j = c, d$, the first order conditions for the NBS are

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{p\left(\frac{(x_1-k_1^*)^{1-R}}{1-R}\right)+(1-p)\left(\frac{(x_2-k_2^*)^{1-R}}{1-R}\right)}{p\left(\frac{(k_1^*)^{1-R}}{1-R}\right)+(1-p)\left(\frac{(k_2^*)^{1-R}}{1-R}\right)}\right) = \frac{(x_1-k_1^*)^{-R}}{(k_1^*)^{-R}}$$

and

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{p\left(\frac{(x_1-k_1^*)^{1-R}}{1-R}\right)+(1-p)\left(\frac{(x_2-k_2^*)^{1-R}}{1-R}\right)}{p\left(\frac{(k_1^*)^{1-R}}{1-R}\right)+(1-p)\left(\frac{(k_2^*)^{1-R}}{1-R}\right)}\right) = \frac{(x_2-k_2^*)^{-R}}{(k_2^*)^{-R}}$$

Simplifying, these two equations reduce to

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{p(x_1-k_1^*)^{1-R}+(1-p)(x_2-k_2^*)^{1-R}}{p(k_1^*)^{1-R}+(1-p)(k_2^*)^{1-R}}\right) = \left(\frac{k_1^*}{x_1-k_1^*}\right)^{R}$$

and

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{p(x_1-k_1^*)^{1-R}+(1-p)(x_2-k_2^*)^{1-R}}{p(k_1^*)^{1-R}+(1-p)(k_2^*)^{1-R}}\right)=\left(\frac{k_2^*}{x_2-k_2^*}\right)^R$$

Since for the case at hand we know that $k_2^*=\left(\frac{x_2}{x_1}\right)k_1^*$, the second equation can be dispensed with entirely, and we can simply locate the solution using the following single equation in a single unknown

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{p(x_1-k_1^*)^{1-R}+(1-p)(x_2-\left(\frac{x_2}{x_1}\right)k_1^*)^{1-R}}{p(k_1^*)^{1-R}+(1-p)(\left(\frac{x_2}{x_1}\right)k_1^*)^{1-R}}\right)=\left(\frac{k_1^*}{x_1-k_1^*}\right)^R$$

While this is a rather formidable looking equation, fortunately it reduces quite simply to something extremely simple and intuitive. Firstly, since $x_2-\left(\frac{x_2}{x_1}\right)k_1^*=\left(\frac{x_2}{x_1}\right)(x_1-k_1^*)$, we have

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{p(x_1-k_1^*)^{1-R}+(1-p)(\left(\frac{x_2}{x_1}\right)(x_1-k_1^*))^{1-R}}{p(k_1^*)^{1-R}+(1-p)(\left(\frac{x_2}{x_1}\right)k_1^*)^{1-R}}\right)=\left(\frac{k_1^*}{x_1-k_1^*}\right)^R$$

that is

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{(x_1-k_1^*)^{1-R}\left(p+(1-p)\left(\frac{x_2}{x_1}\right)^{1-R}\right)}{(k_1^*)^{1-R}\left(p+(1-p)\left(\frac{x_2}{x_1}\right)^{1-R}\right)}\right)=\left(\frac{k_1^*}{x_1-k_1^*}\right)^R$$

Cancelling the common term $\left(p+(1-p)\left(\frac{x_2}{x_1}\right)^{1-R}\right)$, we have

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{x_1-k_1^*}{k_1^*}\right)^{1-R}=\left(\frac{k_1^*}{x_1-k_1^*}\right)^R$$

which with a minimal amount of effort reduces to

$$k_1^*=\alpha x_1 \tag{8}$$

that is, we have

Theorem 3 *If both the creator and the distributor have CRRA utility functions with the same degree of relative risk aversion, then the NBS selects the royalty contract that stipulates that the creator receives a proportion of sales revenue equal to his relative bargaining power.*

In particular, note that the fixed proportion of sales revenue that the NBS contract locates is independent of R and p. If we take the most traditional type of royalty contract that is signed between creators and publishers[8] , we would find that the fixed proportional sharing rule is

[8] See Baumol and Heim (1967) for the case of book publishing, and Towse (2001) for the case of music. Towse (2001), chapter 5, indicates that royalty percentages for performace rights in music in England (dealt with through copyright collectives) may be as high as 50% (although it is unclear if the base is revenue or profit or some other measurement). In Spain, the creators of musical compositions receive a percentage of income generated y performance rights that is between 85 and 90%.

generally set at something around 10 percent, that is, it corresponds to $\alpha = 0.1$, or the distributor has a relative bargaining power that is 9 times greater than that of the creator. Is this a reasonable approximation to what real-life intuition suggests? Consider, for example, the well known fact that when the time between offers collapses to 0 in the alternating offers bargaining game of Rubinstein (see Rubinstein (1982)), its sub-game perfect equilibrium converges to the NBS defined by bargaining powers equal to the inverse of each participants' intertemporal discount factor (see, for example, Binmore (1992) for a simple proof of this). If the distributor is a large corporation (for example, a record company), it is reasonable to assume that its discount factor is equal to the interest rate, and it is also reasonable to assume that an individual creator has a discount factor that is greater than the interest rate (that is, an individual creator is certainly more impatient than a copyright collective).

Naturally, there are other factors that may influence the value of the relative bargaining powers, and that may provide a far better defence for the creator having a relative bargaining power that is 9 times smaller than that of the distributor. For example, one thing that we have completely omitted in the preceding analysis is the differences that occur both in independent wealth and in outside opportunities for each party (that is, the relative need of each for the distribution contract to exist). For the case of performance rights, as we have argued above, successfully managing the copyright to a musical composition implies such high transactions costs that is really not feasible to administer outside of a copyright collective, and so indeed it certainly does seem reasonable that the creator's need is greater, which would be consistent with a value of α that is significantly less than one half. On the other hand, copyright collectives are not really corporations, but rather a large group of copyright holders acting together, in which case they probably will not want to excersise excessive bargaining power against individual members, which may in turn explain why individual royalty percentages for the case of copyright collectives may be as high as 50% (see Towse (2001)).

4 Conclusions

In this paper we have considered, in a very simple setting, efficient royalty contracts between the creator of a cultural work and its distributor. Perhaps one of the most important examples of such contracts are those signed between copyright holders (by defect, creators) and copyright collectives. Since a general characteristic of such contracts is that they have fixed proportional sharing rules (the percentage share of sales revenue is independent of the final value of sales revenue), our interest has been in considering the efficiency of such contracts.

We find that, under certain quite general conditions (increasing and concave utility for both creator and distributor), there is always at least one efficient contract that does specify a fixed proportion sharing rule, and furthermore, if marginal utility with zero wealth is strictly finite, and if the creator has non-negative prudence and the distributor's prudence is not greater than twice his absolute risk aversion, then there is exactly one such contract. For these cases, however, it seems very unlikely that the single contract (out of an infinite set) will be consistently selected, and so we have carried on to consider the case for which all efficient contracts involve a fixed proportion sharing rule.

The relevant case has been shown to correspond to constant and common relative risk aversion. For this case, independently of all parameters, all efficient contracts stipulate that the percentage shares of revenue will be independent of the state of nature that emerges. However, we also indicated the rather delicate nature of this result, since if the creator and the distributor have constant but different relative risk aversion, then no efficient contracts (except the two origins) are consistent with fixed proportion sharing rules.

We have also pointed out that, if we must assume that one of the creator or distributor is more risk averse, then it is surely most logical that the most risk averse of the two is the creator. This view is upheld by noting that it is common to see distributors offering insurance to creators through royalty advance contracts. However, such a situation also implies that any contract that stipulates that the royalty percentage increases with the amount of sales revenue, as is also often the case in real world contracts, must be inefficient (at least for the case of constant relative risk aversion). Hence, clearly, there is a very large set of real-world contracts that are currently in use that can be improved upon.

Finally, we have considered the Nash bargaining solution as a mechanism for selection of a single contract from among the efficient set. We noted that, for the case in which all efficient contracts have a fixed proportion sharing rule, the proportional share corresponds to the relative bargaining powers of the parties to the contract. We noted that real world contracts, in which it is frequent to see fixed proportion sharing rules with the creator retaining a share of approximately 10 percent of revenue, may be argued to be efficient given the relative impatience of creators compared to copyright collectives, and given the greater relative need of creators for the contract to exist.

As far as future research is concerned, there are several ways in which the current analysis can be extended and improved. It may be worthwhile to consider a more general set of states, perhaps even a continuous set of states of nature, instead of the simple two dimensional setting used here. On one hand, it is doubtful that further insights will be gained, and it is certainly true that this particular extention will imply a significant increase in mathematical complexity, but on the other hand, this extention will allow real world data to be used and therefore, the theoretical model could be contrasted with valid empirical data. Secondly, as far as the theoretical model itself is concerned, it would certainly be interesting to include differences between the creator and the distributor on dimensions other than their pure roles within the model. Risk free wealth could be included quite simply, and may be a powerful way of explaining bargaining powers. Naturally, it would also be interesting to carry out an empirical study with the objective of seeing if the assumptions of constant and common relative risk aversion, and a relative bargaining power of about 0.1 for creators are reasonable assumptions for the case at hand.

Finally, there is a significant background risk problem that has been completely ignored in our model, in which we have studied the relationship between a creator and a distributor quite independently from all other such contracts. In particular, it is certainly true that the distributor will generally already hold a portfolio of existing contracts with other creators, to which it is considering adding the current contract. It is also true that there are likely to be significant covariance effects that may actually reduce the risk that the distributor faces. Exactly how such background risk issues will effect the characteristics of the efficient contract would be most interesting.

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