Process Spillovers and Growth.

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Abstract

This paper develops a non-Schumpeterian endogenous growth model of R&D in which the firm's free-riding behavior, reinforced by a lack of appropriability in its industry, constitutes a major source of growth in the economy. While models analyzing the interaction between either imitation and innovation or spillovers and innovation have already appeared in the literature, we show how imitation via free-riding behavior and spillovers can mutually promote dynamic competition and hence economic growth. The representative industry, which is of duopolistic market structure, comprises a leader who innovates and a laggard who free-rides by exploiting the source of intra-industry spillover. We find firstly that the innovation strategies of the two firms can be dynamically strategic complements if a large technology gap prevails and, secondly, that there is a case for process reverse engineering as a fall in the level of appropriability results in higher growth.

Keywords: process imitation, innovation, spillovers, technology gap, endogenous growth
JEL Classification Numbers: C7, E0, L1, O3

1. Introduction

One of the prominent features of R&D based endogenous growth models is that the existence of spillovers due to the lack of appropriability and the resulting competition are always detrimental to an economy's growth performance. However, it is difficult to reconcile this prediction with real life facts. According to Kamien and Schwartz (1982), empirical studies on the relationship between market structure and the rate of diffusion of innovations indicate that innovation is positively related to the competitiveness of the industry into which it is introduced. Also, Cohen and Levithal (1989) found that the effect of appropriability on innovative activity is negative and significant, and hence concluded that contrary to traditional results intra-industry spillovers may encourage R&D investments in equilibrium.[2] Clearly, imitation or free-riding behavior driven by the presence of R&D spillovers is a potential source of competitive pressure that deters industry leaders from behaving as monopolists and prompts them to innovate further.

This paper presents a theoretical inspection of the effects of process spillovers on competition in a R&D based endogenous growth model. Our main concern is to characterize the dynamic interaction between innovation and imitation (via free-riding behavior) when spillovers generated by the former activity makes the latter easier. This interaction introduces an element of strategic complementarity between innovative and imitative strategies at the steady state equilibrium. The analytical framework is based on a two-stage noncooperative differential game between two firms; a leader and a follower

[2] They argue that yet another important role of R&D is to enhance the firm's ability to assimilate and exploit existing information. In this paper free-riding via reverse engineering is made possible by spillovers which facilitate the follower's absorption and learning of the leader's technology.

in a representative industry.[3] In this setting, we examine the long-term behavior of each firm given the dynamics of their technology gap. In contrast to previous studies, we show that, owing to the presence of spillovers from innovation, the dynamic best response of the leader facing imitation[4] is to innovate further rather than to dwell on its short-run higher profits. Since the aggregate rate of innovation is given by the sum of the firms' specific innovation or imitation, the economy's growth steady state rate depends on the growth rate of innovation which, in turn, depends on the growth rate of imitation. In the transitional dynamics, the same findings are observed for large technology gaps. Moreover, it is possible that an increase in appropriability reduces innovation and hence growth.

One important assumption of the model is the duopolistic structure of the representative industry. We therefore define the market configuration of the industry in terms of the relative technology gap.[5] While Traca and Reis (2003) compare the market configuration stability between the symmetric and asymmetric cases, we show that along the transitional dynamics the technology gap growth path is stable and we therefore infer that the underlying market configuration in our model is stable.[6]

One noteworthy implication of this paper is that it helps to provide some economic basis for the phenomenon of reverse engineering. The model demonstrates that at least for the case of process reverse engineering, less appropriability is better as it promotes growth. Handa (1995) in a study of legal implications of reverse engineering

[3] Unlike Traca and Reis (2003), in which there is no a priori difference between the leader and the laggard apart from the initial technology gap, we emphasize the fact that the leader only innovates, while the follower, benefiting from a relatively larger spillover free-rides on the leader.
[4] We do not distinguish between imitation and free-riding behavior in this paper.
[5] This implies that a larger gap means that the leader controls a larger market share and hence the market configuration is asymmetric. The symmetric case is when competition is neck and neck.
[6] One possible line of defense in favor of the duopolistic structure is that the barriers to entry or fixed costs ensure that only two firms can thrive in the market at any point in time.

concluded that the Canadian Copyright Act is juridicially underdeveloped and too uncertain to provide solutions. In yet another contribution on legal implications of reverse engineering, Samuelson and Scotchmer (2002) argued that restrictions on reverse engineering ought to be imposed only if they are justified in terms of the specific characteristics of the industry and their economic effects. We see the process of reverse engineering as a key determinant of innovation in the long-run.

The rest of the paper is organized as follows. In section 2 we provide a brief overview of the literature. In section 3 we present a dynamic general equilibrium model featuring innovation, imitation and spillovers. In section 4, we present our results and section 5 contains some brief concluding remarks.

2. Related Work

The main point of departure of our model from the traditional R&D based endogenous growth framework is that it is competitive rather than monopolistic behavior at the R&D level which generates growth. The prevailing paradigm is based on Schumpeter's idea of creative destruction and models within such frameworks are often referred to as Schumpeterian models. Aghion and Howitt (1992) show, in a model of vertical innovations, that the prospect for more future research discourages current research by threatening to destroy the rents created by such research . Similar views are shared by Grossman and Helpman (1991a; 1991b) and Barro and Sala-i-Martin (1995).

Some of these studies highlight the role of ongoing product upgrading and product cycles[7] in characterizing the steady state equilibrium. In particular, the firm holding the state-of-the-art, that is the one with the lowest price adjusted quality, acts as a

[7] This is due to Vernon (1966).

monopolist in the representative industry.[8] The firms in the latter play a Bertrand game competing on price adjusted quality and such a structure by design leads to a monopolistic market configuration at any point in time.[9] Another consequence of the homogenous Bertrand game assumption is that imitation can be carried out only by relatively lower-cost firms, while successful innovations lead to instantaneous leapfrogging. They show that in general three equilibria exist: the monopolist is a low-cost imitator, the monopolist is a leader who has regained its lead from a low-cost imitator and the monopolist is an innovator who has leapfrogged the leader. Connolly (1997, 1999, 2001) building on the above models, introduces the idea of reverse engineering and learning-to-learn as sources of technological diffusion in North-South trade. Despite her emphasis on the importance of imitation in the transitional dynamics, the concept of creative destruction is still inherent to her analysis.

More recently the question of whether easier imitation of technological leaders is necessarily bad for growth has been given increasing attention. Aghion et al (1997, 2001) have shown that when imitation occurs, leaders tend to innovate further to escape competition or to reestablish their lead. Such models following their spirit have been referred to as non-Schumpeterian models. One of the motivations of this framework is that when the doctrine of creative destruction in R&D based endogenous growth models is applied to real life, it leads to counterfactual predictions. It is therefore possible that there exists some missing link which can explain the empirical failure of Schumpeterian

[8] The idea of quality ladder is also pioneered by these authors and a higher step of the ladder is reached only if another firm leapfrogs the current leader.

[9] Though in Segerstom's (1991) model there can be two firms producing the state-of-the-art, the market structure is still monopolistic since those firms would form a coalition.

models. In this paper, we show that dynamic interactions between firms in an economy represent a potential candidate for that missing element.

Meanwhile, other studies working in the non-Schumpeterian paradigm, have looked into the relationship between product market competition and growth. Aghion et al (1997, 2001), using a model in which R&D incentives occur only in three possible states, found that innovative incentives are higher in the neck and neck state. However, their models do not incorporate the externalities generated by innovative activities. Our paper is closest to Traca and Reis (2003) who, in a model of duopolistic competition within the endogenous growth paradigm, show that spillovers raise the rate of innovation as they spur a source of competitive pressure on the leader. Although our approach is similar to theirs, our model differs from theirs in non-trivial ways. First, spillovers in our model are heterogeneous as the follower who reverse engineers the leader's innovation benefits from larger externalities than the leader. Such heterogeneity is not addressed in their paper. Secondly, we show that the results remain robust in the transitional dynamics as long as the technology gap is large enough and that the policy maker can control for the nature of the dynamic equilibrium by choosing the level of the industry's appropriability.[10]

Another non-Schumpeterian model with no spillovers is developed by Mukoyama (2003) who shows that subsidizing imitation might increase the economy-wide rate of technological progress and that competition and growth might be positively correlated.

3. The Model

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3.1 Overview

We consider a model with n goods, n industries with 2 firms each, and infinitely lived identical consumers. The latter face two optimization problems: temporal and intertemporal utility maximization. Preferences across goods and time are logarithmic. In the intertemporal problem the consumer chooses the optimal labor supply and consumption (or expenditure) for each period. The remaining income is invested in the industries' R&D. To simplify the model we make the following assumptions.[11] Firstly, we normalize expenditures to allow the rate of return to capital (savings of agents) to be constant and equal to the exogenously given discount rate. Moreover, we also assume that the risk of any firm is idiosyncratic and that the stock market values the firm so that its expected rate of return equals the risk free interest rate.[12] Secondly, we assume that labor supply is perfectly elastic.[13]With the optimal amount to be spent in each period chosen, the representative agent can thus derive his demand function for each industry from his temporal optimization problem.

On the production side, the industry demand is derived from the consumer problem and taking such schedule as given, the duopolists in the representative industry compete in Cournot fashion to choose their respective output and research intensities (which are innovation rate for the leader and imitation rate for the laggard). If the productivity of one firm is higher than its competitor, then the former is the leader and the latter is the follower. A further qualification to the structure of our representative industry is the existence of heterogeneous intra-industry spillovers. Being an innovator,

[11] These do not lead to loss of generality in our propositions.
[12] This is similar to Grossman and Helpman (1991).
[13] This follows from Aghion et al (2001).

the leader does not benefit much in terms of externality from the follower[14] , while the follower which practises process reverse engineering benefits more than the leader.

The choices of the firm's variables which are quantity and research intensities are done sequentially. We therefore use backward induction in a two-stage noncooperative game setting to formulate the firms' optimum behavior. In the second stage the firms play a Cournot game to determine their respective quantities. Thus their respective profits as functions of their productivity[15] levels can thus be derived. In the first stage, the leader (follower) plays a differential game to choose their optimal innovation (imitation) time path taking the technology gap (ratio of their productivity levels) dynamics as their state variable. The open-loop Nash equilibrium is then found.[16] Since innovation and imitation are the only variables growing in the economy, the steady state growth rate is determined by the growth rates of those variables. Dynamic reaction functions are used to derive results. Effects of changes in appropriability and growth are analyzed. Finally the path of the technology gap is derived and some stability conditions are imposed.

3.2 Formal Model

Consumers

Let n , C_t , L_t , Q_{it} , R_t and W_t be the number of industries, the consumption of the representative agent, his labor supply,quantity produced in the industry i for i = 1,2...,n,

[14] We assume that the value of the positive externality accruing to the leader is small but non-zero since there might be some heavily located facilities which are inherent to the setting up of a firm and that there might be some interactions among workers.
[15] Productivity is defined in terms of the per unit cost as in Traca and Reis (2003).
[16] An open-loop Nash equilibrium is found when a competitor takes his rival's reaction function solely as a function of time in his dynamic optimization problem. Essentially, there is only one decision node.

the interest rate and the wage rate respectively at time t. Then the intertemporal preference of the agent can be written as

$$U \equiv \int_0^\infty e^{-\rho t}\left(u(C_t) - L_t\right)dt, \rho > 0 \quad \text{where } \rho \text{ is the discount rate} \qquad (1)$$

$$u(C_t) \equiv \ln(C_t) \qquad (2)$$

The intertemporal utility maximization problem results in (i) $W_t = 1$ and (ii) $R_t = \rho$ after normalization.[17] The temporal consumer preference is given by

$$u(C_t) \equiv \sum_{i=1}^{n} \ln(Q_{it}) \text{ for all t} \qquad (3)$$

The static utility maximization problem results in the industry demand curve[18]

$$Q_{it} = \frac{M}{P_{it}} \qquad \text{where M = 1/n} \qquad (4)$$

Producers

[17] Let consumer's wealth at time t be $\ln(C_t)$, P_t be the price of consumption and $P_t C_t = 1$ due to normalization, then we have
$H = \ln(C_t) - L_t + \lambda_t(R_t A_t + W_t L_t - P_t C_t)$
$dH/dC_t = 0$ implies $1/P_t C_t = \lambda_t$, but since $P_t C_t = 1$, $\lambda_t = 1$
$dH/dL_t = 0$ implies $W_t \lambda_t = 1$ and hence $W_t = 1$
Also $d\lambda_t / dt = \rho\lambda - \lambda R_t$, but since $\lambda_t = 1$, $d\lambda_t / dt = 0$ or $\rho - R_t = 0$ and therefore $R_t = \rho$
[18] Static utility maximization leads to

$$\underset{Q_i}{Max}\; u(C_t) \equiv \sum_{i=1}^{n} \ln(Q_{it}) \text{ s.t}$$

$$\sum_{i=1}^{n} P_i Q_i = I$$

and since Income = Expenditure = 1, the constraint becomes

$$\sum_{i=1}^{n} P_i Q_i = 1$$

The logarithmic assumption leads to the following demand curve $Q_{it} = \frac{1}{nP_{it}}$

Given the industry demand (4) each firm will choose its respective optimal production q_{ijt} such that[19]

$$\sum_{j=1}^{2} q_{ijt} = Q_{ijt}$$

We assume that firm 1 is the leader and firm 2 is the follower. Each firm's production function is given by

$q_j = A_j L_j$ for j =1,2 (5)

It can easily be inferred from (5) that the per unit cost of each firm is given by W^*/ A_j where W^* is the economy level wage rate. Also, due to our assumption $W_t = 1$, the per unit cost becomes

$c_j = 1/ A_j$ (6)

The productivity dynamics is assumed to be given by

$\dot{A}_{jt} = h\Lambda_{jt} L^{R\&D}{}_{jt}$ where h is the R&D productivity (7)

Λ_{jt} is the spillover to firm j and $L^{R\&D}$ is the labor employed in the R&D sector. It is understood from our formulation that each firm operates in two sectors in which it employs labor. Our underlying assumption here is that workers are homogeneous since a constant wage rate ensures that no skill differentials among the workers are observable in the labor market. The term h is the R&D productivity level, which is assumed to be given in the industry. We are therefore left to qualify the spillovers Λ_{jt} which are the underpinnings of our analysis.

[19] In this subsection of the paper, we sometimes omit subscript i for simplicity.

Our definition of spillovers is similar to Cohen and Levinthal (1989) together with some extensions. In particular we define spillovers to include valuable knowledge generated in the research process of the leader and which becomes accessible to the follower if and only if the latter is reverse engineering the innovator's research process. It is important here to note that had the follower not been imitating the leader, the spillover it enjoys would reduce to a small positive number equal to that of the leader (see footnote 13). This also implies that the homogeneous assumption of Traca and Reis (2003) will be a special case of our model. Given this assumption of relatively larger spillovers favoring imitation vis-à-vis innovation, it becomes a better strategy for the follower to imitate by feeding off the leader's innovation at least initially. Thus the follower is necessarily an imitator.

It is also implicit from our assumption that it is process imitation rather than product imitation which takes place in our framework. This also means that the conventional definition of reverse engineering as the decompilation of a finished product in order to gain a better understanding of how it was produced as in Handa (1995) does not fit well into our model. Rather we see reverse engineering as the act of extracting know-how or information from the industry leader through channels like the labor market (turnover in R&D personnel, for example) in order to imitate the latter's process (or cost-cutting) innovations.[20] Hence, unlike Schumpeterian models, spillovers are not regarded as a pure public good since some effort (imitation) is involved in acquiring it. We formally let the spillovers for firm 1 and 2 be

$$\Lambda_{1t} = A_{1t}^{1-\sigma_1} A_{2t}^{\sigma_1}$$

[20] Nevertheless, our definition still belong to a more general class of definitions of reverse engineering.

$$\Lambda_{2t} = A_{2t}^{1-\sigma_2} A_{1t}^{\sigma_2} \qquad (8)$$

where σ_1 , σ_2 are less than ½ and $\sigma_2 > \sigma_1$. We let σ_2 be inversely related to the appropriability level of the industry and we see that this expression will also increase the spillovers enjoyed by the follower. One can also think of it as a tool for the policy maker to regulate or protect patents. The second restriction ensures that imitators enjoy larger spillovers than innovators. The technology gap G_t is given by

$$G_t = \frac{A_{1t}}{A_{2t}} > 1 \qquad (9)$$

where the inequality shows that firm 1 is the leader. Also, the gap dynamics is given by

$$\dot{G}_t = (\alpha_{1t} - \alpha_{2t})G_t \qquad (10)$$

where

$$\alpha_{jt} \equiv \frac{\dot{A}_{jt}}{A_{jt}} \qquad \text{for } j = 1,2$$

α_1 and α_2 are the growth rates of innovation and imitation respectively. This completes the model.

3.3 Solving the Model

We solve the model by backward induction.

Stage 2

Using the inverse demand function (4), firm j's profit maximization problem becomes

$\underset{q_j}{Max}$ $Mq_j/(q_j + q_i) - c_j q_j$, j =1,2 and i is not equal to j (11)

The Cournot Nash quantity for firm j is given by

$q_j = (Mc_i)/(c_j + c_i)^2$, j =1,2 and i is not equal to j (12)

The profit function for firm j is given by

$\Pi_j = (Mc_i^2)/(c_j + c_i)^2,\ j = 1,2$ and i is not equal to j (13)

Proof: See Appendix

So far we have not provided a rationale for why we depart from the prevailing literature which uses a Bertrand differentiated product competition rather than a Cournot competition. We offer two justifications as to why this assumption suits our purpose better here. First, the homogeneous Cournot assumption by design implies that the only way for the leader (follower) to increase his market share is to increase (decrease) the cost differential which is given by the technology gap as shown in the next set of equations. Therefore the possibility of product innovation is ruled out in this setting and hence our model necessarily implies that all the imitation and innovation occur at the process level. Both Aghion et al. (2001) and Traca and Reis (2003) consider only process imitation in the former case and process spillovers in the latter case but yet they use a formulation (Bertand differentiated) in which both product and process innovations are possible. Secondly, our assumption allows the two firms to compete in both the product market and the R&D sector even for the case of product homogeneity unlike the Bertrand homogeneous game. This enables us to compare our model directly with the Schumpeterian paradigm (at least at the micro level) without changing the assumption of product homogeneity.

For the sake of simplifying the remainder of the analysis we rewrite the profit functions of (13) and research costs as functions of the technology gap only G_t .We thus have

$\Pi_{1t} = M/(1 + 1/G_t)^2$ (14)

Π_{2t}= M/(1+ G_t)^2 (15)

and

R&D cost of firm 1 = $\alpha_{1t}G_t^{\sigma_1}/h$ (16)

R&D cost of firm 2 = $\alpha_{2t}/hG_t^{\sigma_2}$ (17)

Proof: See Appendix.

Stage 1 (The Open-Loop Formulation)

The pair (θ_1, θ_2) is called an open-loop Nash equilibrium with function θ_j mapping t $\in [0,T)$ to a real number if for each j = 1, 2 , an optimal control path α_j(.) of the problem below exists and is given by $\alpha_j(t)= \theta_j(t)$.[21] As shown below the optimal control is performed with this definition as a basis. It is also important to note that, unlike the case described by Vencatachellum (1998), the open-loop Nash equilibrium in our model does not coincide with the myopic strategy whereby the firm does not take into account the productivity of its rival while choosing its optimal path.[22] Firm 1's dynamic optimization problem is given by

$$V_1 = \max_{\alpha_1} \int_0^{\infty} e^{-\rho t} \left[M\left(1+\frac{1}{G_t}\right)^{-2} - \frac{\alpha_{1t}G_t^{\sigma_1}}{h} \right] dt$$

s.t $\dot{G}_t = (\alpha_1 - \alpha_2)G_t$, G_0 is given, $G_T \geq 0$ as T $\rightarrow \infty$ and $\alpha_{1t} \geq 0$ (18)

[21] The general case is formulated by Dockner et al. (2000). We assume an open-loop equilibrium as in Peretto (1996) since we are unable to find a closed-form solution to analyze the properties of the model for a closed-loop or Markov perfect equilibrium. In principle, if the objective function is of linear quadratic form, the closed-loop equilibrium can be found by setting the Hamilton Jacobi Bellman (HJB) equation.

[22] This special case arises since the Hamiltonian of one firm is linear and separable in its rival's stock of human capital and hence, the latter term vanishes at the first order condition.

Firm 2's dynamic optimization problem is given by

$$V_2 = \max_{\alpha_2} \int_0^{\infty} e^{-\rho t} \left[M(1+G_t)^{-2} - \frac{\alpha_{2t}}{G_t^{\sigma_2} h} \right] dt$$

s.t $\dot{G}_t = (\alpha_1 - \alpha_2) G_t$,G_0 is given, $G_T \geq 0$ as $T \to \infty$ and $\alpha_{2t} \geq 0$ (19)

The Hamiltonian function for firm 1 can be written as

$$H_1 = M\left(1+\frac{1}{G_t}\right)^{-2} - \alpha_{1t}.\frac{G_t^{\sigma_1}}{h} + \lambda_{1t}(\alpha_{1t} - \alpha_{2t})G_t \tag{20}$$

The first order conditions are

$$\frac{dH}{d\alpha_{1t1}} \leq 0, \alpha_{1t} \geq 0$$

$$\lambda_{1t} G_t - \frac{G_t^{\sigma_1}}{h} \leq 0, \alpha_{1t} \geq 0, (\lambda_{1t} G_t - \frac{G_t^{\sigma_1}}{h})\alpha_{1t} = 0 \text{ (Kuhn Tucker conditions)} \tag{21}$$

$$\dot{\lambda}_{1t} = \rho\lambda_{1t} - \frac{2M}{G_t^2}\left(1+\frac{1}{G_t}\right)^{-3} + \frac{\alpha_{1t}\sigma_1 G_t^{\sigma_1 - 1}}{h} - \lambda_{1t}(\alpha_{1t} - \alpha_{2t}) \tag{22}$$

Transversality conditions

$\lim_{T\to\infty}$ $e^{-\rho t}$ λ_{1T}=0 if G_T >0

$\lim_{T\to\infty}$ $e^{-\rho t}$ $\lambda_{1T} \geq 0$ if G_T =0

Combining and from Kuhn Tucker again we have, $\lim_{T\to\infty}$ $e^{-\rho t}$ λ_{1T} G_T = 0 (23)

The Hamiltonian function for firm 2 can be written as

$$H_2 = M(1+G_t)^{-2} - \alpha_{2t} \cdot \frac{1}{hG_t^{\sigma_2}} + \lambda_{2t}(\alpha_{1t} - \alpha_{2t})G_t \tag{24}$$

The first order conditions are

$$\frac{dH}{d\alpha_{2t}} \le 0, \alpha_{2t} \ge 0$$

$$-\lambda_{2t}G_t - \frac{1}{hG_t^{\sigma_2}} \le 0, \alpha_{2t} \ge 0, (-\lambda_{2t}G_t - \frac{1}{hG_t^{\sigma_2}})\alpha_{2t} = 0 \text{ (Kuhn Tucker conditions)} \quad (25)$$

$$\dot{\lambda}_{2t} = \rho\lambda_{2t} + 2M(1+G_t)^{-3} - \frac{\alpha_{2t}\sigma_2}{hG_t^{\sigma_2+1}} - \lambda_{2t}(\alpha_{1t} - \alpha_{2t}) \quad (26)$$

Transversality conditions

$\lim_{T\to\infty} e^{-\rho t} \lambda_{2T}=0$ if $G_T > 0$

$\lim_{T\to\infty} e^{-\rho t} \lambda_{2T} \ge 0$ if $G_T = 0$

Combining and from Kuhn Tucker again we have, $\lim_{T\to\infty} e^{-\rho t} \lambda_{2T} G_T = 0$ (27)

4. Results

4.1 Steady State

We now characterize the steady state of the economy by finding the steady state of one industry and assuming that all other industries are operating at their respective steady state levels. As in Traca and Reis (2003), we found that at the steady state the leader's rate of innovation equals the follower's rate of imitation. Formally, the dynamic equilibrium in our model has the following properties at the steady state:

(i) At the steady state $\alpha_1 = \alpha_2$.

(ii) $X_0 > \rho$ is a sufficient condition for both α_1 and α_2 to be positive at time t = 0 where $X_0 \equiv 2Mh(G_0+1)^{-3}$ and G_0 denotes G_t at time t = 0.

(iii) The solution G_s to $\max\{X_sZ_s, X_sY_s\} = \rho$ is a stagnation steady state with neither innovation nor imitation if $\min\{X_sZ_s, X_sY_s\} < \rho$

where

$Z_s \equiv G_s^{\sigma_2+1}$

and

$Y_s \equiv G_s^{2-\sigma_1}$

where G_0 denotes G_s at the steady state.

Proof: See Appendix

We can now formulate the main propositions of this paper.

4.2 Imitation and Appropriability in the transitional dynamics

We saw that rates of innovation and imitation are equal at the steady state. As a corollary, we also have that an increase in imitation by the follower leads to an increase in innovation by the leader and since their constant growth rate is the only variable growing in the representative industry we infer that such an increase would raise the economy's growth rate. Hence by increasing imitation, laggards put pressure on the industry leader to innovate more and it is this interaction which, in turn, drives the economy's engine of growth. Yet another corollary of the previous subsection is that at the steady state the growth rate of the technology gap is zero; thus the market configuration is stable at the steady state. Similar results were also obtained by Traca and Reis (2003). We next show that these results remain robust in the transitional dynamics under some assumptions on the gap.

Proposition 4.2.1(Imitation)

For large technology gaps, imitation and innovation are strategic complements in their transitional dynamics; that is an *increase in imitation by the laggard leads to an increase in innovation by the leader if the technology gap is large enough.*

Proof:

From (A21), we know that

$$\sigma_2\alpha_{1t} = X_t Z_t - \rho \tag{28}$$

Given X_t and Z_t from (ii) and (iii)

(28) can be rewritten as

$$\sigma_2\alpha_{1t} = \frac{2MhG_t^{\sigma_2+1}}{\left(1+G_t\right)^3} - \rho \tag{29}$$

We want to find the effect of a change in α_{2t} on α_{1t} .Since from (A24) we know that α_{2t} and G_t are negatively related for all $G_t>2$, it suffices to show that α_{2t} and G_t are negatively related. From (29),

$$\frac{d\alpha_{1t}}{dG_t} = \frac{2MhG_t^{\sigma_2}}{\sigma_2(G_t+1)^6}\left[(G^3+3G^2+3G+1)(\sigma_2+1)-(3G^3+6G^2+3G\right] \tag{30}$$

The above term is negative if and only if

$$\left(2-\sigma_2\right)G_t \ +\left(3-3\sigma_2\right) > \frac{3\sigma_2}{G_t} + \frac{\sigma_2+1}{G_t^{\ 2}} \tag{31}$$

Given our earlier parameter restrictions on (8) and (9), (31) is true by definition. Hence by chain rule, the impact of α_{2t} on α_{1t} is positive.▪

Proposition 4.2.1 shows that through the establishment of an important long-run relationship between imitation and innovation the steady state result also holds during the transitional dynamics under the assumption of large technology gaps. In this dynamic game between the two firms with imitation and innovation as strategic variables, we observe that at any point in time an increase in imitation rate by the follower will prompt the leader to increase his innovation rate as long as the latter has a significant advantage over the former. Since there are only two variables growing in the representative industry[23] and the dynamic relationship between them is positive, it must be the case that the dynamic interactions between those firms will make the industry competitive at all times. In other words, process imitation creates a source of competitive pressure which deters the leader from maximizing short-run monopoly profits but rather "forces" him to innovate further.

The restriction of large gaps (G_t>2 as shown in the appendix) ensures that no leapfrogging takes place in our model. For small technology gaps, the relationship between imitation and innovation is no longer positive as the leader does not have enough productivity lead and anticipates that the imitator might close the gap by feeding off the intra-industry spillovers. Our assumption of large gaps therefore rules out such possibilities. Moreover, Aghion et.al's (2001) findings that both firms have lower R&D incentives in the special case when the gap equals zero give some insights to our assumption.

The proof for proposition 4.2.1 is instructive since crucial to its construction is the mechanism which explains the above result. This is due to the fact that we derived the effect of imitation on the technology gap first, followed by finding the effect of the latter on innovation and eventually inferred the result by simple chain rule. Hence, imitation first reduces the gap (assuming the gap is not too narrow), and the leader receiving the signal that his technological advantage is shrinking puts in effort to restore his lead. We can also see that without the restriction of large technology gaps, an increase in the technology gap can potentially increase imitation and that the best response of the leader then would be to reduce his innovation to prevent the imitator from benefiting from the

[23] Recall that the economy consists of n such prototypes and hence the economy should be growing at the rate of growth of the representative industry.

positive externalities generated by his activities. Thus in this case the follower is considered as too close to the leader for the latter to allow him to free-ride. Our restriction rules out the occurrence of the above scenario. While proposition 4.2.1 makes a strong case for reverse engineering, it also strengthens the results of most non-Schumpeterian models.

Proposition 4.2.2 (Spillovers)

In the transitional dynamics, due to the existence of a non-Schumpeterian effect, an industry with a relatively lower degree of appropriability does not necessarily grow at a slower rate; that is, an increase in the ease of spillovers or an improvement in the reverse engineering environment does not necessarily lead to a fall in the rate of innovation.

Proof:

From (31), we have

$$\alpha_{1t} = X_t Z_t \left(\frac{1}{\sigma_2} \right) - \rho \left(\frac{1}{\sigma_2} \right) \qquad (32)$$

We observe that there are two components affecting α_{1t}.Since ρ is constant, the effect of a change in σ_2 on α_{1t} due to the second component of the RHS of (32) is positive. The effect of σ_2 on α_{1t} due to the first component depends on the effect of σ_2 on X_tZ_t which in turn depends on the time path of G_t .The latter in equilibrium will depend on σ_2 ,σ_1 and t. Since explicit an expression for the time path of G_t cannot be found, we consider two cases.

Case (i) $\frac{d}{d\sigma_2}\left(\frac{X_t Z_t}{\sigma_2} \right) < 0$

In this case the first component is the usual Schumpeterian effect and the second component, which is unambiguously positive, is our postulated non-Schumpeterian effect.

Case (ii) $\frac{d}{d\sigma_2}\left(\frac{X_t Z_t}{\sigma_2}\right) \geq 0$

In this case we only have a non-Schumpeterian effect.

But (i) and (ii) imply that the effect of an increase in σ_2 *on* α_{1t} *is at least ambiguous. Therefore we conclude that an increase in* σ_2 *does not necessarily reduce* α_{1t}. ▪[24]

Proposition 4.2.2 shows that laws prohibiting process reverse engineering or policies designed to mitigate factors promoting it are not justifiable at least from the economic growth perspective. It demonstrates the impact of a decrease in appropriability (increase in σ_2) on the leader's Nash equilibrium value of innovation. We find that a higher patent protection rate in an industry does not increase innovation unambiguously since there exists a non-Schumpeterian effect working in the opposite direction of the Schumpeterian effect. Thus the heterogeneity in spillovers with a higher weight given to the one accrued by the imitator allows us to separate the impact of a general industry-level externality (σ_1) and externalities which enhance imitative behavior (σ_2). This result gives some theoretical insight into Cohen and Levinthal's (1989) empirical studies in which they conclude that the negative incentives effect of spillovers and hence the advantages of policies designed to mitigate them might not be as great as supposed. It also helps shed some light on the recent law debates surrounding the advantages and disadvantages of legalizing the act of reverse engineering.

Proposition 4.2.3 (technology gap)

If the level of appropriability in an industry is bounded from below[25] *, then* $S_t \leq 0$ *is a necessary and sufficient condition for the stability of the dynamic system in our model,*

[24] Note that for case (i), there will always exist a level of the discount factor which would ensure a non-Schumpeterian effect.

where $S_t \equiv \left(\frac{\sigma_2 + 2}{\sigma_2}\right) G_t^{\sigma_2+\sigma_1-1} + \left(\frac{\sigma_2 - 1}{\sigma_2}\right) G_t^{\sigma_2+\sigma_1} + G_t - \left(\frac{3-\sigma_1}{\sigma_1}\right)$ *; that is this condition ensures the existence of a Saddle-path to the steady state. Moreover this leads to a stable market configuration.*

Proof: See Appendix

Proposition 4.2.3 shows that as long as the specified condition on the technology gap is satisfied, the latter will always converge and the dynamic system is stable. Although it is a prima facie that this condition is merely to satisfy some technical conditions in control theoretic models, two important corollaries arise from it. First, as described in footnote 5, since the market share of a firm depends on the size of the technology gap, the degree of competition or monopolization will be determined by the dynamics of the latter. Now, since from proposition 4.2.3 we know that the path of technology gap converges and is stable, we can infer that the market structure or configuration is stable in the transitional dynamics under the given condition.[26]

The second interesting corollary to emerge from the above proposition is that the condition given will impose on the gap an upper bound to which its path converges asymptotically. This means that a gap which is very large is not feasible in our model since there will be excessive free-riding from imitation. If we had allowed for the latter to occur by relaxing the condition in proposition 3.2.3, it would have been feasible that the leader might find it optimal not to innovate at all at some point in time. One can also observe that by the definition of G_t the lower bound for technology gap is 1. Now, given the latter and the upper bound restriction of proposition 3.2.3, we conclude that the

[25] This is equivalent to saying that σ_2 is bounded from above.

[26] Of course the rate of growth of technology gap at the steady state is zero.

technology gap and hence the market configuration is bounded in this model. Thus, the symmetric case where competition is neck and neck (when the gap tends to zero) and pure monopolization (when the gap tends to infinity) is never attained in the transitional dynamics. Hence, there is always (at any point in time) a follower who will prompt the leader to innovate further in such a market configuration and this will lead to higher growth. It is also noteworthy that the above phenomenon might be due to increasing returns on the R&D when the gap is large.[27] According to Glass (2000), an important factor in Japan's recent economic slowdown is that they have exhausted all imitation possibilities as they move closer to the world's technology frontier.

Proposition 4.2.4 (Policy Implication)

If the level of appropriability in an industry and the technology gap are bounded from below, the equilibrium with innovation and imitation as dynamic strategic complements exists, is unique, and is stable; that is, by choosing the spillover parameter, the policy maker can ensure the existence and uniqueness of a steady state with Saddle-path where imitation and innovation are positively related assuming that the gap is not too narrow.

Proof: See Appendix.

Proposition 4.2.4 shows that the policy maker can maneuver the nature of dynamic equilibrium by choosing the level of appropriability (inversely related to σ_2). To see this, let us think of the game in our model as a three stage game with a "pseudo" first stage in which the policy maker chooses $\sigma_2 - \sigma_1$ once and leaves it there.[28] Thus, by the same logic of backward induction described in earlier sections, the latter, acting like a "Stackelberg leader", can ensure the stability of equilibrium with a large technology gap

[27] See Peretto (1996) for further comment.

[28] The choice is a one-shot action in this stage as compared to the second stage in which the choices are sequential.

and in which imitation and innovation are dynamic strategic complements. It is therefore possible that the policy maker can promote growth by choosing a lower bound level of appropriability (upper bound to σ_2)

It is also important to note that, as in proposition 4.2.1, this proposition also depends on the assumption of large technology gaps (G_t>2); for if it does not hold, the proof shows that the equilibrium with imitation and innovation as dynamic strategic substitutes can be stable. One possible explanation for our result not holding for narrow technology gaps, aside from the one given earlier (that the follower is "too close" to the leader for the latter to allow him to continue free-riding), is that the follower's marginal imitation induces relatively lower change in his market share as compared to when the gap is large. In economic terms, there is decreasing returns to scale as the technology gap narrows. This fact is also confirmed by empirical findings in the literature. (see Glass (2000), Peretto (1996)).

5. Concluding Remarks

We have presented an analytical model that deals with process imitation and spillovers in a non-Schumpeterian framework. Our motivation comes mainly from an apparent lacuna in existing non-Schumpeterian models in showing the interrelation between process imitation and spillovers and their impact on growth. Moreover, existing Schumpeterian models, lack adequate empirical evidence to explain growth using the concept of creative destruction. Indeed most of these studies rely heavily on the price undercutting mechanism of the homogeneous Bertrand game. We demonstrate without relaxing the assumption of product homogeneity that competitive behavior can still prevail by using a Cournot quantity competition setting. Two main factors drive competitive behavior in the long-run; firstly, imitation by the follower and, secondly, spillovers occurring due to a lack of appropriability.

The paradigm proposed in this paper can offer a basis for understanding how the dynamic strategic interactions between two firms with a technology gap can determine the economy's growth rate. In particular, imitation acts as a spur by putting pressure on the industry leader to innovate further and this drives the economy's engine of growth. Furthermore, this research can contribute to the literature on "The Law and Economics of Reverse Engineering" (See for example, Samuelson and Scotchmer, 2002) by providing some economic grounds in favor of process reverse engineering. In this regard, it demonstrates the existence of a non-Schumpeterian element in the innovator's best response function. One immediate policy implication of our model is that laws and regulations which hinder process imitation might not always be a good thing in an industry characterized by spillovers.

An obvious extension of our analysis would be to consider alterations to the duopolistic structure. While we measure market configuration (or relative monopolistic structure vis-à-vis competition) only by the technology gap between the two firms, we do not allow for entry (see footnote 6). However, we also believe that more firms entering the industry could only mean more competition and this would provide a case for non-Schumpeterian models. Further research might address the issue of entry in industries with more than two firms, or consider closed-loop games formulation rather than the open-loop case as proposed by Traca and Reis (2003) and in this paper.

6. Appendix

6.1 Derivation of the second stage quantity, profit, and R&D cost functions

In this section we derive (12)-(17).

From (11), firm j's problem is given by

$$\underset{q_j}{Max} \quad Mq_j/(q_j + q_i) - c_j q_j \;, \; j = 1,2 \quad \text{and} \quad i \text{ is not equal to } j \tag{11}$$

FOC for firm j is given by

$$(q_j + q_i)M - q_jM - c_j(q_j + q_i)^2 = 0 \tag{A1}$$

By symmetry we have,

$$(q_i + q_j)M - q_iM - c_i(q_i + q_j)^2 = 0 \tag{A2}$$

Simplifying gives

$$q_iM - c_j(q_j + q_i)^2 = 0 \tag{A3}$$

$$q_jM - c_i(q_i + q_j)^2 = 0 \tag{A4}$$

Solving (A3) and (A4) simultaneously gives (12)

Replacing (12) for both firms in (4) gives P_t

$$P_t - c_j = c_i/(c_j + c_i)^2 \tag{A5}$$

Thus the profit for firm j is given by

$$(P_t - c_j)\, q_j = \Pi_j \tag{A6}$$

(A6) verifies (13)

Using (6) and (13) for firm 1 we have

$$\Pi_1 = M(1/A_2)^2/(1/A_2 + 1/A_1)^2 \tag{A7}$$

$$\Pi_2 = M(1/A_1)^2/(1/A_2 + 1/A_1)^2 \tag{A8}$$

(9), (A7) and (A8) give (14) and (15)

Combining (7) and the identity in (10), we have

$$L_{jt}^{R\&D} = \frac{\alpha_{jt} A_{jt}}{h\Lambda_{jt}} \tag{A9}$$

Now using (A9) , (8) ,(9) combined with the fact that wage rate =1 give (16) and (17)

6.2 Proof for (i) – (iii) of the steady state.

(i) Assuming $\alpha_{1t} > 0$ in the case where the first order condition of the control variable is satisfied with equality and using (21), we have

$$\lambda_{1t} = \frac{G_t^{\sigma_1 - 1}}{h} \tag{A10}$$

Taking the derivative of (A10) w.r.t time we have

$$\dot{\lambda}_{1t} = \left(\frac{\sigma_1 - 1}{h}\right) G_t^{\sigma_1 - 1} \left(\alpha_{1t} - \alpha_{2t}\right) \tag{A11}$$

Combining (22), (A10) ,(A11) by substituting X_t ,Y_t , Z_t where needed and simplifying, we have

$$(\sigma_1 - 1)(\alpha_{1t} - \alpha_{2t}) = \rho + \sigma_1 \alpha_{1t} - (\alpha_{1t} - \alpha_{2t}) - X_t Y_t \tag{A12}$$

We prove (i), that is, that $\alpha_{1t} = \alpha_{2t}$ by contradiction.

Suppose not, then there are two possibilities : (a) $\alpha_{1t} > \alpha_{2t}$,(b) $\alpha_{1t} < \alpha_{2t}$

Case (a) If $\alpha_{1t} > \alpha_{2t} \geq 0$, then $G_t \to \infty$ as $T \to \infty$

Now since $X_t Y_t$ depend on G_t, it can be shown using L'Hopital rule that as $G_t \to \infty$ $X_t Y_t \to 0$[29]

Using the above fact and simplifying (A12) gives

$$\alpha_{2t} = \frac{-\rho}{\sigma_1} \tag{A13}$$

But this is a contradiction since for all $\rho > 0$ and $\sigma_1 > 0$, $\alpha_{2t} < 0$ contradicts $\alpha_{1t} > \alpha_{2t} \geq 0$

Case (b) If $\alpha_{2t} > \alpha_{1t} \geq 0$, then $G_t \to 0$ as $T \to \infty$

[29] Proof can be provided upon request.

Now since X_tY_t depend on G_t, it can be shown using L'Hopital rule that as $G_t \to 0$, $X_tY_t \to \infty$.

Using the above fact and simplifying (A12) gives

$$-\alpha_{2t}\sigma_1 = \rho - X_t Y_t \quad \text{as } X_tY_t \to \infty \tag{A14}$$

But this implies that $\alpha_{2t} \to \infty$ which gives a contradiction to $\alpha_{2t} \in [0,\infty)$ for all t.

Since (a) and (b) are not possible, it must be that $\alpha_{1t} = \alpha_{2t}$. ▪

For consistency sake we show that the proof can also be derived from firm 2's behavior.

Assuming $\alpha_{2t} > 0$ in the case where the first order condition of the control variable is satisfied with equality and using (25), we have

$$\lambda_{2t} = \frac{-1}{hG_t^{\sigma_2+1}} \tag{A15}$$

Taking the derivative of (A15) w.r.t time we have

$$\dot{\lambda}_{2t} = \left(\frac{(\sigma_2+1)(\alpha_{1t}-\alpha_{2t})}{hG_t^{\sigma_2+1}}\right) \tag{A16}$$

Combining (26), (A15) ,(A16) by substituting X_t ,Y_t , Z_t where needed and simplifying, we have

$$(\sigma_2+1)(\alpha_{1t}-\alpha_{2t}) = -\rho - \sigma_2\alpha_{2t} + (\alpha_{1t}-\alpha_{2t}) + X_t Z_t \tag{A17}$$

We prove (i), that is, that $\alpha_{1t} = \alpha_{2t}$ by contradiction.

Suppose not, then there are two possibilities : (a) $\alpha_{1t} > \alpha_{2t}$,(b) $\alpha_{1t} < \alpha_{2t}$

Case (a) If $\alpha_{1t} > \alpha_{2t} \geq 0$, then $G_t \to \infty$ as $T \to \infty$

Now since X_tZ_t depend on G_t, it can be shown using L'Hopital rule that as $G_t \to \infty$ $X_tZ_t \to 0$

Using the above fact and simplifying (A12) gives

$$\alpha_{1t} = \frac{-\rho}{\sigma_2} \tag{A18}$$

But this is a contradiction since for all $\rho>0$ and $\sigma_1>0$, $\alpha_{1t}<0$ contradicts $\alpha_{1t} > \alpha_{2t} \geq 0$

Case (b) If $\alpha_{2t} > \alpha_{1t} \geq 0$, then $G_t \to 0$ as $T \to \infty$

Now since X_tZ_t depend on G_t , it can be shown using L'Hopital rule that as $G_t \to 0$ $X_tZ_t \to \infty$.

Using the above fact and simplifying (A12) gives

$$-\alpha_{1t}\sigma_2 = \rho - X_t Z_t \quad \text{as } X_tZ_t \to \infty \tag{A19}$$

But this implies that $\alpha_{1t} \to \infty$ which yields a contradiction to $\alpha_{2t} > \alpha_{1t}$ and $\alpha_{2t} \in [0, \infty)$ for all t.

Since (a) and (b) are not possible, it must be that $\alpha_{1t} = \alpha_{2t.}$ ▪

(ii) We now show that as assumed by (i), α_1 and α_2 are indeed positive at the steady state.

(A14) can be rewritten as

$$\alpha_{2t}\sigma_1 = X_t Y_t - \rho \tag{A20}$$

By visual inspection of (A20), we see that $X_0 Y_0 > \rho$ as initial condition at t = 0 is a sufficient condition for α_2 to be positive in the initial state.

Also, (A19) can be rewritten as

$$\alpha_{1t}\sigma_2 = X_t Z_t - \rho \tag{A21}$$

By visual inspection of (A21), we see that $X_0 Z_0 > \rho$ as initial condition at t = 0 is a sufficient condition for α_2 to be positive in the initial state.

But since both Y_0 and Z_0 are > 1 by definition, it must be that

$X_0 > \rho$ is a sufficient condition for both α_1 and α_2 to be positive at time t = 0.▪

(iii) Those conditions can easily be inferred from (A20) and (A21).

6.3 Proof for negative relationship between α_{2t} and G_t for large G_t.

Rewriting (A20), we have

$$\sigma_1\alpha_{2t} = \frac{2MhG_t^{2-\sigma_1}}{(1+G_t)^3} - \rho \qquad \text{(A22)}$$

$$\frac{d\alpha_{2t}}{dG_t} = \frac{2MhG_t^{1-\sigma_1}}{\sigma_1(G_t+1)^6}\left[(G^3+3G^2+3G+1)(2-\sigma_1)-(3G^3+6G^2+3G\right] \qquad \text{(A23)}$$

The above term is negative if and only if

$$(1+\sigma_1)G_t \;\; +3\sigma_1 > \frac{3-3\sigma_1}{G_t} + \frac{2-\sigma_1}{G_t{}^2} \qquad \text{(A24)}$$

Given our earlier parameter restrictions on (8) and (9), (A24) is true for all G_t>2.▪

6.4 Proof of Proposition 4.2.3

Using (A12) and (10) by letting $\dot{G}_t \;/ G_t = g_t$, we have

$$\sigma_1 g_t = \rho + \sigma_1\alpha_{1t} - X_t Y_t \qquad \text{(A25)}$$

Using (A17) and (10) by letting $\dot{G}_t \;/ G_t = g_t$, we have

$$-\sigma_2 g_t = \rho + \sigma_2\alpha_{2t} - X_t Z_t \qquad \text{(A26)}$$

From (A25), we have

$$\alpha_{1t} = (-\rho + \sigma_1 g_t + X_t Y_t)/\sigma_1 \qquad \text{(A27)}$$

From (A26), we have

$$\alpha_{2t} = (-\rho - \sigma_1 g_t + X_t Z_t)/\sigma_2 \qquad \text{(A28)}$$

Solving (A27), (A28) and (10) by letting $\dot{G}_t / G_t = g_t$, we have

$$g_t = \frac{X_t Z_t}{\sigma_2} - \frac{X_t Y_t}{\sigma_1} + \left(\frac{\sigma_2 - \sigma_1}{\sigma_2 \sigma_1}\right)\rho \tag{A29}$$

$$\frac{d\dot{G}_t}{dG_t} = \frac{2MhG_t^{2-\sigma_1}}{(1+G_t)^4}\left\{(1+G_t)\left[\left(\frac{\sigma_2+2}{\sigma_2}\right)G_t^{\sigma_2+\sigma_1-1} - \left(\frac{3-\sigma_1}{\sigma_1}\right)\right] - 3\left[\frac{G_t^{\sigma_2+\sigma_1}}{\sigma_2} - \frac{G_t}{\sigma_1}\right]\right\} + \frac{(\sigma_2-\sigma_1)\rho}{\sigma_2\sigma_1} \tag{A30}$$

For $\sigma_2 - \sigma_1 \rightarrow 0$ (which can hold only if σ_2 is bounded from above), the RHS of (A30) is negative if and only if

$$S_t \equiv \left(\frac{\sigma_2+2}{\sigma_2}\right)G_t^{\sigma_2+\sigma_1-1} - \left(\frac{1-\sigma_2}{\sigma_2}\right)G_t^{\sigma_2+\sigma_1} + G_t - \left(\frac{3-\sigma_1}{\sigma_1}\right) \leq 0 \tag{A31}$$

This completes the proof.▪

<u>6.5 Proof of Proposition 4.2.4</u>

Using (A29) and the fact that at the steady state $g_t = 0$, we have

$$\rho(\sigma_2 - \sigma_1) = X_t Y_t \sigma_2 - X_t Z_t \sigma_1 \tag{A32}$$

Re-arranging by substituting the expressions for X_t, Y_t, and Z_t,we have

$$\frac{(1+G_t)^3\rho}{2Mh} = \frac{G_t^{2-\sigma_1}\sigma_2 - G_t^{\sigma_2+1}\sigma_1}{\sigma_2 - \sigma_1} \tag{A33}$$

Since $G_t^{2-\sigma_1}\sigma_2 > G_t^{\sigma_2+1}\sigma_1$ (given our early parameter restrictions σ_2 ,σ_1<1/2 , $\sigma_2 > \sigma_1$ and G_t>1), we observe from (A33) that as $\sigma_2 - \sigma_1 \rightarrow 0$ (assuming that if σ_2 is bounded from above), there exists some $\underline{G_t}$ such that the RHS of (A33) > LHS of (A33). Hence, we have $\underline{G_t}$ where $1 < \underline{G_t} < \infty$ such that RHS> LHS. Now it can also be shown that both the RHS and LHS of (A33) are monotonically increasing and convex for our early parameter restrictions σ_2 ,σ_1<1/2 , σ_2 >σ_1 and G_t>1[30]. Thus by visual inspection of (A33), we see that as $G_t \to\infty$, the LHS of (A33) > RHS of (A33) since the power of the terms in G_t of the LHS are always higher than that of the RHS. Therefore, given the monotonocity of the LHS and the RHS, we infer that there exists some $\overline{G_t}$ such that the LHS of (A33) > RHS of (A33). Hence we have $\overline{G_t}$ where $1 < \underline{G_t} < \overline{G_t} < \infty$ such that LHS> RHS. Using (A33), we define a function F_t given by

$$F_t(G_t) = \frac{(1+G_t)^3 \rho}{2Mh} - \frac{G_t^{2-\sigma_1}\sigma_2 - G_t^{\sigma_2+1}\sigma_1}{\sigma_2 - \sigma_1} \qquad \text{(A34)}$$

As noted above, for some G_t , RHS> LHS and for some G_t, LHS> RHS. Thus for some G_t , F_t is positive and for some G_t , F_t is negative. It well-known from the Weierstrass Intermediate Value Theorem that if a continuous function on an interval is sometimes positive and sometimes negative, it must be zero at some point. Let this point be G_t^* . This proves the existence of a fixed point such that $1 < \underline{G_t} < G_t^* \overline{< G_t} < \infty$. The proof for uniqueness follows from the monotonocity of both sides off (A33).

We now prove for Saddle path stability assuming $\sigma_2 - \sigma_1 \to 0$ (this can hold if σ_2 is bounded from above).

We observe that for earlier parameter restrictions σ_2 ,σ_1<1/2 , σ_2 >σ_1 and G_t>1, the fourth term of (A31) is larger than its first term. A sufficient condition for stability is therefore that the second term is larger than the third term. This is true if and only if

[30] Proof can be provided upon request.

$$\left(\frac{1-\sigma_2}{\sigma_2}\right)G_t^{\sigma_2+\sigma_1} > G_t \qquad \text{(A35)}$$

$$\text{or } \; G_t > \left[\frac{1-\sigma_2}{\sigma_2}\right]^{\frac{1}{1-\sigma_2-\sigma_1}} \qquad \text{(A36)}$$

This establishes a lower bound for the technology gap which will ensure stability. Hence if the gap is large enough ($G_t > \left[\frac{1-\sigma_2}{\sigma_2}\right]^{\frac{1}{1-\sigma_2-\sigma_1}}$) and the level of appropriability is bounded from below (σ_2 is bounded from above and thus, $\sigma_2 - \sigma_1 \rightarrow 0$), the system is stable. We now show that the equilibrium with innovation and imitation as dynamic strategic complements is stable.

Recall from Proposition 4.2.1 that if $G_t > 2$, innovation and imitation are strategic complements in their transitional dynamics. In other words, we need the lower bound on the technology gap to be larger than 2, that is

$$G_t > \left[\frac{1-\sigma_2}{\sigma_2}\right]^{\frac{1}{1-\sigma_2-\sigma_1}} > 2 \; \text{ or} \qquad \text{(A37)}$$

$$\frac{1-\sigma_2}{\sigma_2} > 2^{1-\sigma_2-\sigma_1} \qquad \text{(A38)}$$

But (A38) holds by definition given our parameter restrictions. Hence, the path on which innovation and imitation are strategic complements is a Saddle path. This completes the proof. ▪

7. References

Aghion ; Howitt P, 1992. A Model of Growth through Creative Destruction. *Econometrica,* 60, 323-351.

Aghion P; Harris C; Howitt P; and Vickers J, 1997.Competition, Imitation and Growth with Step by Step Innovation. *European Economic Review,* 41,771-782.

Aghion P; Harris C; Howitt P; and Vickers J.,2001. Competition, Imitation and Growth with Step by Step Innovation. *Review of Economic Studies,* 68, 467-492.

Barro R. and Sala-i-Martin X,1995. Economic Growth. *McGraw-Hill, New York*.

Cohen W; Levinthal D,1989.Innovation and Learning: The Two Faces of R&D. *Economic Journal,* 99, 569-596.

Connolly M,1997. The Role of Imitation and Trade in Technological Diffusion. *Working Paper No 97-25*, Duke University, Economics Department.

Connolly M,1999. North- South Technological Diffusion. A New Case for Dynamic Gains for Gains from Trade. *Working Paper No 99-08* ,Duke University, Economics Department.

Connolly M,2001. The Dual Nature of Trade: Measuring its Impact on Imitation and Growth. *Working Paper* No 97-34, Duke University, Economics Department.

Dockner E.J; Long V.N; Gerhard S; Jorgensen S, 2000. Differential Games in Economics and Management Science. *Cambridge University Press.*

Glass A.J, 2000. Imitation as a stepping stone to Innovation. *Working Paper*, Department of Economics, Ohio State University.

Grossman G; Helpman H,1991. Quality Ladders in the Theory of Growth. *Review of Economic Studies*, 58, 43-61.

Grossman G; Helpman H, 1991. Quality Ladders and Product Cycles.*The Quarterly Journal of Economics*, 106, 557-586.

Handa S, 1995. Reverse Engineering of Computer Programs under Canadian Copyright Law. *Mc Gill Law Journal*, 40 621-690.

Kamien M ; Schwartz N, 1982. Market Structure and Innovation. *Cambridge University Press* .

Mukoyama T, 2000. Innovation, Imitation and Growth with Cumulative Technology. *Journal of Monetary Economics,*50,361-380.

Peretto P, 1996. Sunk Costs, Market Structure and Growth. *International Economic Review,* 37, 895-923.

Romer P, 1994. New Goods and the Welfare Costs of Trade Restrictions. *Journal of Development Economics,* 43, 312-330.

Samuelson Pamela; Scotchmer S, 2002. The Law & Economics of Reverse Engineering. *Yale Law Journal*, (forthcoming).

Schumpeter J.A, 1934. "The Theory of Economic Development" *Oxford University Press.*

Sergerstrom Paul.S, 1991. Innovation, Imitation and Economic Growth, *The Journal of Political Economy*, 99, 807-827.

Traca D; Reis A, 2003. Spillovers and the Competitive Pressure for Long-Run Innovation, *Unpublished.*

Vencatachellum D, 1998.Endogenous Growth with Strategic Innovations, *Journal of Economic Dynamics and Control,* 23, 233-254.

Vencatachellum D, 1998.A Differential R&D Game: Implications for Knowledge- Based Growth Models.*Journal of Optimization Theory and Applications*, 96,175-189.

Vernon R, 1966. International Investment and International Trade in the Product Cycle. *Quarterly Journal of Economics,* 80, 190-207.