

REVENUE SHARING AS COMPENSATION FOR COPYRIGHT HOLDERS

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ABSTRACT. Essential inputs are an important topic of debate for economics. One common essential input is intellectual property, in the form of either patents or copyrights, which the producers of goods and services for final consumption must necessarily purchase from the input supplier. The ensuing monopoly power of the input supplier leads in many cases to controversial outcomes, in which social inefficiencies can occur. In much of the literature on the economics of intellectual property, it is assumed that the right holder is remunerated either by a fixed payment or by a payment that amounts to an additional marginal cost to the user, or both. However, in some significant instances in the real-world, right holders are constrained to use (or may choose to use) a compensation scheme that involves revenue sharing. That is, the right holder takes as remuneration a part of the user's revenue. In essence, the remuneration is set as a tax on the user's revenue. This paper analyses such remuneration mechanisms, establishing and analysing the optimal tax rate, and also the Nash equilibrium tax rate that would emerge from a fair and unconstrained bargaining problem. The second option provides a rate that may be useful for regulatory authorities. The model is calibrated against a (hypothetical) scenario in which the copyright holders in music are paid a regulated share of the revenue of music radio stations, a topic that is presently at the fore-front of the economics of copyright pricing.

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1. INTRODUCTION

In much of the literature on contracts for the use of intellectual property as an essential input to a production process, the royalty compensation is set as either a fixed cost to the user or as an element of additional marginal cost (see, for example, Kamien and Tauman (1986), Wang (1998), Fosfuri and Roca (2004), and Sen (2005)). This is reasonable for some, but not all settings. As far as the specific case of remuneration for copyright holders goes, while the royalty earnings of artists, singers and literary authors are often set as above (a royalty payment per unit of sales), in other cases the royalty payment that is allocated to the right holder as compensation is set as a tax upon the user's revenue.¹ This, for example, is the typically the case for music that is played publicly on the radio, which is the motivating example for the current paper. The literature on revenue sharing contracts is quite extensive, although it concentrates almost exclusively upon correcting for incentive effects along the supply chain (see, for example, Cachon and Lariviere (2005), and Wang et al. (2004)). There is also a small literature that examines revenue sharing in the video rental market (Dana and Spier (2001), Mortimer (2008)).

Attempts to examine theoretically the optimal royalty tax for the right holder to set are rare. Michel (2006) considers the case of a copyright holder supplying music to a record label in exchange for a share in revenue. While the optimal revenue share (from the point of view of the copyright holder) is not discussed, Michel does consider the Nash bargaining revenue share. However, the main thrust of the Michel paper is to consider the comparative

¹The two royalty schemes may or may not be equivalent. Say total revenue is px , where p is the unit price and x is the level of sales. A tax of t on revenue gives the copyright holder income of tpx , and leaves the user with income of $(1-t)px$. On the other hand, if the royalty is a payment per unit sold, of say r , then the copyright holder gets a payment of rx , and the user has income of $(p-r)x$. If the price p were constant, then the two are equivalent if $r = tp$. However, in the more realistic case of price varying with sales, the two are not generally equivalent.

statics of copyright piracy and Internet file sharing upon the bargained contract. Although Michel is able to study certain comparative static features of the Nash bargaining solution, the complexity of the environment assumed by Michel does not allow a closed form solution to the problem to be found. Second, Marchese and Ramello (2011), in the context of religious messages, study a game in which a copyright holder (the Church) supplies an essential input to a distributor while simultaneously competing in the output market. However, the objective of the right holder in that paper is not to maximise income, but rather to maximise diffusion.

Perhaps the paper that is most related to the present work is Watt (2010), who directly considers the remuneration of the copyright holders in the music that is played on music radio stations.² In that paper, a totally different methodology, based on the Shapley value, is employed to find a fair revenue tax rate for paying copyright holders. Interestingly, in spite of the very different methodology of the solution, the general conclusions from that paper (that the currently used tariffs appear to be too low) coincides with the conclusions from the present paper.

In the present paper I put forward an analysis of the remuneration that the supplier of a copyrighted work that is used as an input into a production process that is not directly undertaken by the copyright holder himself. The analysis here is only concerned with the case in which the payment to the copyright holder is restricted to being a tax on the revenues of the producer that uses the copyright as an input, and while the model of the present paper can certainly be more widely applied, it is intended to shed some light upon cases such as those of the tariff for compensating copyright holders in the music that is played on music radio stations. The copyright holder in the present paper acts with the objective of maximising income,

²See also Audley and Boyer (2007).

and so the type of scenario here is likely to be relevant to a large number of other settings in which essential inputs are involved. Above all, aside from considering the unrestricted monopoly price that the copyright holder would optimally charge, the paper looks closely at the Nash bargaining outcome, and attempts to provide useful solutions to the dilemma faced by regulators of such transactions – how can a fair, and also relatively non-complex, price for the use of the copyrighted work be determined?³

The paper continues as follows. In the next two sections, the underlying assumptions for the model that is used are stated and analysed. A few simple but important results concerning the tax rate that the copyright holder would set when acting optimally, are generated. In section 4 we look at the way in which the total surplus from the relationship is shared when the tax on revenue is set optimally (in an unconstrained market). Then, in section 5 we assume that the copyright holder is constrained to a “fair” tax rate. This is modelled using a Nash bargaining game. All of the analysis up to and including section 5 is carried out under the assumption of no fixed costs in the production process of the copyright user. In section 6 this critical assumption is changed, and the effect of fixed costs are explicitly examined. In section 7 the model is calibrated with an illustrative example concerning the supply of copyrighted music to music radio stations. While the numbers used are only meant to be illustrative, it is hoped that they are not too wildly distant from some real-world situations. Finally, section 8 concludes.

Throughout the paper, only principal mathematical formulas and derivations are shown in the main text. The more tedious workings are relegated to the appendix. The paper is also very dependent on graphs to show the

³I stress that, in order to be of any practical use, the tariff that is suggested for compensating the copyright holders should be non-complex. In the paper we will see that many different tariffs can be easily constructed, but if the tariff is overly complex, it becomes rather useless for the practical purposes of offering guidance to real-world regulators.

main results. The only really conditioning assumptions that are used from the outset are linear demand for the user’s output, and constant marginal costs for the user’s production process. However, given those two assumptions we are able to generate some precise formulas for the main dependent variables. These formulas can be easily graphed using any simple mathematical software package, and the graphs that appear in the paper are done in precisely that way. Showing the actual and exact graphical representations of the formulas saves on the tedious mathematical process of analysing their slopes (whether they are positive or negative, whether they are increasing or decreasing, whether they are concave or convex, etc.).

2. MODELLING ASSUMPTIONS

We shall assume that the setting is one of a single supplier of a copyrighted work⁴ and a single user of that copyrighted work in a production process. The copyrighted work that is supplied is essential to the user’s production process, in the sense that without access to the copyrighted input, the user is unable to produce any output for consumers. We assume that the user acts as a monopolist in the market for the final good or service that he produces, which we denote by x , and likewise the supplier of the copyrighted input acts as a monopolist in the supply of the copyrighted work. For reasons that are determined exogenously (e.g. the prevailing legal environment), the remuneration of the supplier is set as a tax on the revenue that is generated by the user.⁵ In the interests of tractability of the model that follows, we assume that the demand for the final consumption good produced by the

⁴The term “supplier” is used liberally. More precisely, the supplier of the copyrighted work could be a collecting society that represents a whole group of copyright holders, and the copyrighted input itself could be an entire repertory of individual elements.

⁵Actually, it can be shown that a revenue tax will always dominate a per-unit royalty (see Marchese and Ramello, 2011). It is also true that for each feasible revenue sharing rule, there is an equivalent profit sharing rule. Revenue sharing as opposed to profit sharing avoids the complications of asymmetric information regarding costs.

user is linear, that is the price at which the good is sold is given by⁶

$$p(x) = 1 - bx$$

and that the user's production process is also linear (i.e. it is characterised by constant marginal cost), with marginal cost equal to c , where at all times $c < 1$. For the initial analysis, the user is assumed to face no fixed costs, and her objective is to maximise monetary profits. The issue of fixed costs, which is of great importance to the relationship in question, will be discussed below in a separate section.

We are only interested in the case in which the copyrighted work is already in existence (and so there are no fixed costs of supplying it, or more to the point, any fixed costs have already been incurred, and are sunk), and units of it can be produced at no cost what-so-ever. This contrasts with what happens with other inputs, which are supplied according to some sort of strictly increasing cost function. Of course, for the case of intellectual property generally it is reasonable to assume that the marginal cost of supplying access is zero. For that reason, in the model below the copyright holder is assumed to face no costs at all in the supply of the copyrighted input to the user, and so her objective is to maximise income.⁷

3. ANALYSIS OF THE OPTIMAL REVENUE TAX RATE

The compensation for the copyright holder is a revenue tax,⁸ and so for any given choice of output x , assuming that the tax is set at t , the user ends

⁶Note that the demand vertical intercept has been set to 1. This is without loss of generality, and simply sets the units of measurement for the problem.

⁷The assumption that both the copyright holder and the copyright user maximise their monetary profits is of course equivalent to assuming that both are risk neutral. In fact, the environment is assumed to be risk free. This is done in order to simplify the analysis as much as possible, given the objective of finding a simple, workable, and useful sharing rule.

⁸Of course this is exactly the same as a tax on price (an ad valorem tax), at least in the context of the present paper. The equivalence is due to the assumption of a demand curve, and a single price for all customers.

up with a profit of

$$\begin{aligned}\pi(t) &= (1-t)(1-bx)x - cx \\ &= [(1-t)(1-bx) - c]x\end{aligned}\quad (1)$$

The copyright holder would get an income of

$$R(t) = t(1-bx)x \quad (2)$$

Once the tax rate has been set, the user will then freely choose his level of output, x , to maximise his profits. Maximising $\pi(t)$ with respect to x gives

$$x^*(t) = \frac{(1-t) - c}{2b(1-t)} \quad (3)$$

The optimal level of output is a decreasing and concave function of the tax rate:

$$x^{*'}(t) = -\frac{c}{2b(1-t)^2} < 0 \quad x^{*''}(t) = -\frac{c}{b(1-t)^3} < 0 \quad (4)$$

Our principal analysis at this point is of the copyright holder's income function. We are interested in considering the value of t that will maximise it, subject to the user's profit remaining non-negative.

It is useful to separate the analysis of $t = 1$ from the analysis of $t < 1$. Clearly, from (3), if $t = 1$, the optimal level of output is not defined. However, there is a correspondence between $c = 0$ and $t = 1$. The reason for this is the following. Assume that $c = 0$. Then, if $t < 1$, the optimal level of output would be $\frac{(1-t)}{2b(1-t)} = \frac{1}{2b}$. What about if $t = 1$? To find out what the optimal output would be we need to analyse the optimal level of output in limit as t approaches 1. But using L'Hôpital's theorem, it turns out that

$$\lim_{t \rightarrow 1} \frac{(1-t)}{2b(1-t)} = \frac{-1}{-2b} = \frac{1}{2b}$$

Thus, independently of what level of t is used, when $c = 0$ the user will set output at $\frac{1}{2b}$. But this implies that the total amount of revenue earned is a constant, independent of t , equal to $(1 - b\frac{1}{2b}) (\frac{1}{2b}) = \frac{1}{4b}$. Since, with $c = 0$,

the user does not alter output (or revenue) in response to a change in the tax rate, the best choice of the copyright holder is clearly to set the tax rate as high as possible in this case, and so we get $t^*(c)|_{c=0} = 1$. With this tax rate, the user produces a strictly positive level of output, and generates a strictly positive level of revenues, which the copyright holder then takes entirely as the royalty payment. The user ends up with 0, and thus is indifferent between producing optimally and not producing at all. Given this, we shall only be concerned from now on with scenarios of $c > 0$. Since it is also necessary that $c < 1$, in all that follows we will only be concerned with $0 < c < 1$.

The non-negativity assumption on the user's profit implies that there is a maximum tax rate that the copyright holder cannot exceed. To calculate this maximum, consider again the user's profit for any tax rate t , assuming of course that the user sets his output choice optimally given that tax rate;

$$\pi(t) = [(1-t)(1-bx^*(t)) - c]x^*(t)$$

The effect of a change in t is given by the first derivative of $\pi(t)$, which is

$$\begin{aligned} \pi'(t) &= [(1-t)(1-bx^*(t)) - c]x^{*'}(t) \\ &\quad - [(1-bx^*(t)) + (1-t)bx^{*'}(t)]x^*(t) \\ &= x^{*'}(t)[(1-t)(1-bx^*(t)) - c - (1-t)bx^*(t)] \\ &\quad - (1-bx^*(t))x^*(t) \\ &= x^{*'}(t)[(1-t)(1-2bx^*(t)) - c] - (1-bx^*(t))x^*(t) \end{aligned}$$

But, think about the first term on the right-hand side. Recalling that⁹

$$x^*(t) = \frac{(1-t) - c}{2b(1-t)}$$

⁹For the following analysis, simply assume $t < 1$. This will be shown to be the case shortly.

we have

$$\begin{aligned} (1-t)(1-2bx^*(t)) - c &= (1-t) \left(1 - 2b \left(\frac{(1-t) - c}{2b(1-t)} \right) \right) - c \\ &= (1-t) \left(1 - \left(\frac{(1-t) - c}{(1-t)} \right) \right) - c \end{aligned}$$

Giving a common denominator to the second term, and then simplifying yields

$$(1-t) \left(\frac{(1-t) - ((1-t) - c)}{(1-t)} \right) - c = c - c = 0$$

Thus it turns out that

$$\pi'(t) = -(1 - bx^*(t))x^*(t) < 0$$

Theorem 1. *The maximum tax rate that the copyright holder can set is $\bar{t} = 1 - c$.*

Proof. The fact that the user's profit is decreasing in t implies that the maximum tax rate that can be set, \bar{t} , satisfies

$$\pi(\bar{t}) = [(1 - \bar{t})(1 - bx^*(\bar{t})) - c] x^*(\bar{t}) = 0$$

So either $(1 - t)(1 - bx^*(t)) - c = 0$, or $x^*(t) = 0$, or both. The tax rate that would set optimal output to 0 is easily calculated from (3) as

$$\bar{t}_1 = 1 - c$$

while the tax rate that sets $(1 - \bar{t}_2)(1 - bx^*(\bar{t}_2)) - c = 0$ is¹⁰

$$1 - \bar{t}_2 = c$$

But this is the same tax rate that sets optimal output to 0, i.e. $\bar{t}_2 = \bar{t}_1$.

Thus the limit tax rate for the copyright holder is $\bar{t} = 1 - c$. \square

While it is necessary to go through the above analysis of the maximum feasible tax rate, it is also worthwhile to note the following:

¹⁰See section 1 of the appendix for all of the steps used in finding this equation.

Theorem 2. *Assuming $c > 0$, the copyright holder will never want to set the tax rate at (or of course above) the maximum feasible level.*

Proof. We are always assuming $c < 1$, which implies that it is always possible for the activities of the user to generate positive profits, and of course positive revenue. However, if the tax rate on revenue is set at the maximum feasible level identified in Theorem 1, the optimal response of the user is to set output at 0, and thus total revenue generated is also 0. This means that the copyright holder will earn no royalty income at all. However, setting the tax rate at a smaller level, such that now the user does produce a positive level of output, and generates a positive total revenue, will imply a positive level of earnings for the copyright holder. Thus, it will never be optimal for the copyright holder to set the tax rate at the maximum level. \square

Theorem 2 has two implications. First, since we are only concerned with scenarios of $c > 0$, we know that we always have $t^*(c) < 1 - c < 1$. This is important as it implies that we can always safely assume $1 - t > 0$, so that divisions by $1 - t$ (which will frequently be done) are valid. Second, we can always safely ignore the restriction that the optimal tax rate set by the copyright holder should satisfy $t^* \leq 1 - c$, and then go ahead with studying the unconstrained optimisation problem for the copyright holder, which is

$$\max_t R(t) = t(1 - bx^*(t))x^*(t)$$

Substituting (3) into (2), we can then see that the copyright holder's objective function is¹¹

$$\begin{aligned} R(t) &= t(1 - bx^*(t))x^*(t) \\ &= \frac{t((1 - t)^2 - c^2)}{4b(1 - t)^2} \end{aligned} \tag{5}$$

¹¹See section 2 of the appendix for the working.

We would like to maximise (5) with an appropriate choice of t . In order to do that, we firstly need to investigate the shape of this function. The first derivative of $R(t)$ is¹²

$$R'(t) = \left(\frac{1}{4b}\right) \left(1 - \frac{(1+t)c^2}{(1-t)^3}\right) \quad (6)$$

The second derivative is

$$R''(t) = -\left(\frac{c^2}{4b}\right) \left(\frac{t+2}{(1-t)^4}\right) < 0 \quad (7)$$

Since the second derivative is negative, $R(t)$ is concave in t , and so the unconstrained optimal t is found where the first derivative is 0. That is

$$t^* \leftarrow 1 = \frac{(1+t^*)c^2}{(1-t^*)^3} \quad (8)$$

We can analyse this equation in a variety of ways, but let us simply write $f(c, t) = 1 - \frac{(1+t)c^2}{(1-t)^3}$, so that we have $f(c, t^*) = 0$. The analysis of the optimal tax rate can then be done by studying the roots of $f(c, t)$.

Theorem 3. *If $c > 0$, there is a single root of $f(c, t)$, and that root occurs at a t that satisfies $0 < t < 1$.*

Proof. Note the following:

$$\begin{aligned} f(c, 0) &= 1 - c^2 > 0 \\ \lim_{t \rightarrow 1} f(c, t) &= -\infty < 0 \\ \frac{\partial f(c, t)}{\partial t} &= -\frac{c^2(1-t)^3 + (1+t)c^2 3(1-t)^2}{(1-t)^6} \\ &= -2c^2 \frac{(2+t)}{(1-t)^4} < 0 \end{aligned}$$

Thus, the function $f(c, t)$ starts out positive at $t = 0$ and it is negative as t approaches 1. So there is at least one root of $f(c, t)$ for a t between 0 and 1. However, since the slope of the function is everywhere negative, there can only be a single root. \square

¹²The working appears in section 3 of the appendix.

The shape of the function $f(c, t)$ is shown in Figure 1.¹³ The root of the function, t^* , is the unique optimal tax rate for the problem. For example, the graph in Figure 1 corresponds to $c = \frac{1}{2}$, in which case we get $t^* = 0.3106$.¹⁴

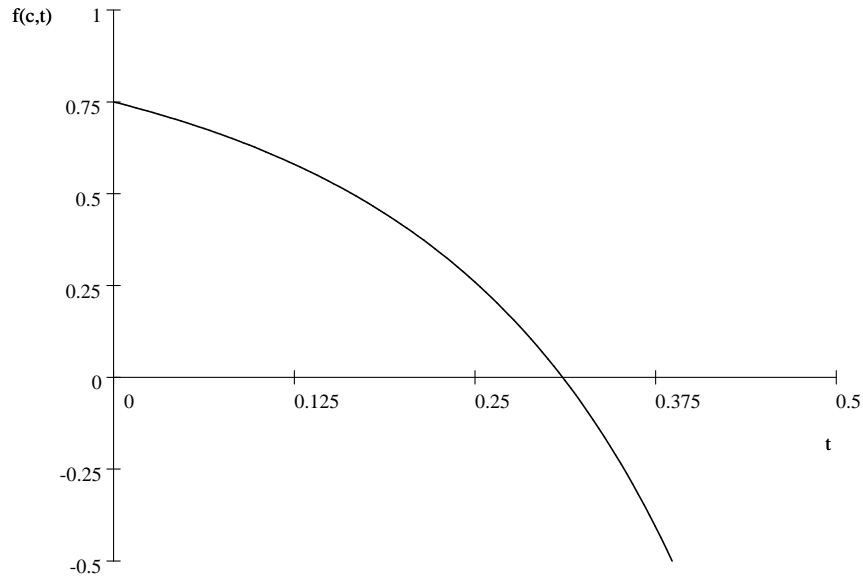


Figure 1: The shape of $f(c, t)$

Given that there is a single root of the function $f(c, t)$, we can find it algebraically as the solution of (8). While not at all easy to do for a third order equation, this is certainly possible. Using the mathematical package DERIVE, the solution turns out to be

$$t^*(c) = 1 + \frac{\sqrt{3}c^{\frac{2}{3}} \left(\sqrt{(c^2 + 27)} - 3\sqrt{3} \right)^{\frac{1}{3}}}{3} - \frac{\sqrt{3}c^{\frac{2}{3}} \left(\sqrt{(c^2 + 27)} + 3\sqrt{3} \right)^{\frac{1}{3}}}{3}$$

¹³Throughout this paper, graphs are used to illustrate the equations derived. All of the graphs are computer generated from the actual equations in the text, and so they are completely accurate representations of each equation. The general characteristics of most of the equations we analyse can also be examined by recourse to the implicit function theorem. However, given that we have the possibility of accurate drawings of the graphs of the equations, which clearly indicate such aspects as slope and curvature, the mathematical analysis will be omitted.

¹⁴Note that for this example, the constraining maximum feasible tax rate is $1 - c = \frac{1}{2}$, the unconstrained optimum is indeed the global optimum for the problem.

The graph of $t^*(c)$, for all values of c between 0 and 1 is given in Figure 2. It is decreasing and convex on the range of feasible values of c .

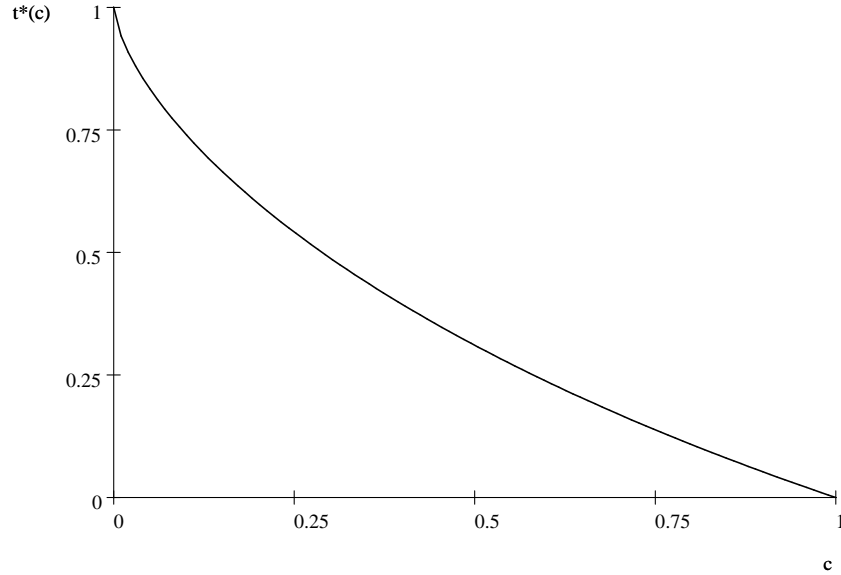


Figure 2: The optimal revenue tax, $t^*(c)$

4. SHARING OF MARKET SURPLUS

In the above, we have simply performed an analysis of the optimal tax rate, from the perspective of the copyright holder. We might also wonder about how the total market surplus is shared between the two parties under such a revenue sharing model.

To analyse surplus sharing, it is useful to express the user's profit as a function of the royalty payment;¹⁵

$$\pi(t) = R(t) \left(\frac{1-t}{t} \right) - x^*(t)c \quad (9)$$

¹⁵The working for this is given in section 4 of the appendix.

Given this, for any tax rate t , the ratio of user profit to copyright holder income is

$$\begin{aligned}\frac{\pi(t)}{R(t)} &= \frac{R(t) \left(\frac{1-t}{t}\right) - x^*(t)c}{R(t)} \\ &= \left(\frac{1-t}{t}\right) - \frac{x^*(t)c}{R(t)}\end{aligned}$$

But since $R(t) = t(1 - bx^*(t))x^*(t)$, we can write

$$\begin{aligned}\frac{\pi(t)}{R(t)} &= \left(\frac{1-t}{t}\right) - \frac{x^*(t)c}{t(1 - bx^*(t))x^*(t)} \\ &= \left(\frac{1-t}{t}\right) - \frac{c}{t(1 - bx^*(t))}\end{aligned}\tag{10}$$

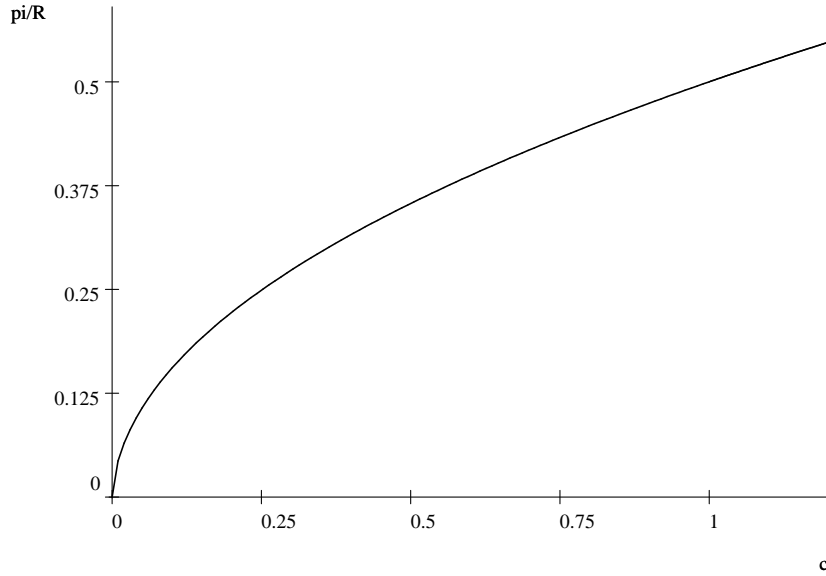


Figure 3: The shape of $\frac{\pi(t^*(c))}{R(t^*(c))}$

Substituting for the optimal output level and simplifying,¹⁶ we get

$$\frac{\pi(t)}{R(t)} = \left(\frac{1-t}{t}\right) \left[\frac{(1-t) - c}{(1-t) + c}\right]\tag{11}$$

¹⁶See section 5 of the appendix for this working.

The graph of $\frac{\pi(t^*(c))}{R(t^*(c))}$ is displayed in Figure 3 as a function of c . It is a strictly increasing and concave graph. At $c = 1$, the height of the graph is 0.5.

From Figure 3, we can establish the following result:

Theorem 4. *For all values of c , the copyright holder earns at least twice as much as the user; $R(t^*) > 2\pi(t^*)$.*

Proof. The proof is evident from the fact that $\frac{\pi(t^*(c))}{R(t^*(c))}$ is located everywhere below the value $\frac{1}{2}$ over the relevant range of values of c . \square

There is a final interesting result that we can prove regarding the way in which total surplus is shared. The result is the following:

Theorem 5. *For any tax rate t , the ratio of copyright holder income to user profit is equal to the elasticity of user profit to the tax rate.*

Proof. In the next section, it will be shown that, for any revenue tax rate, it holds that

$$\pi'(t) = -\frac{R(t)}{t}$$

If we divide both sides of this last expression by $-\pi(t)$, and cross multiply the t , it reads

$$-\frac{t\pi'(t)}{\pi(t)} = \frac{R(t)}{\pi(t)}$$

The left-hand side of this is just the absolute value of the elasticity of the user's profit to the tax rate, $-\varepsilon^\pi(t)$. \square

Of course it is true that this equation is true for all values of t , so it is also true for the optimal tax rate, t^* , so we have

$$\frac{R(t^*)}{\pi(t^*)} = -\frac{t^*\pi'(t^*)}{\pi(t^*)} = -\varepsilon^\pi(t^*)$$

5. THE NASH BARGAINING SOLUTION

The model above has assumed that the copyright holder can set the rate optimally. We noted that when this is done in any scenario in which positive profits can be made ($c < 1$), the copyright holder will end up earning more of the total surplus than will the user. In fact this may not be unreasonable. Firstly, the copyright holder is supplying an essential input to the user's production process, and as such it may be considered reasonable that the copyright holder should earn most of the surplus that is created, since without her input, no surplus at all is created. Secondly, we are allowing the user to act as an unconstrained monopolist in the market for his output, and so it seems fair to allow the same consideration to the copyright holder, and the result that the copyright holder takes the lion's share of the surplus is entirely due to her being able to price monopolistically.

Never-the-less, if the monopoly power of the copyright holder were thought to be giving her an unfair advantage,¹⁷ one could appeal to some other way of establishing the revenue sharing tariff rate. One logical option might be to appeal to the deal that would be struck in an unconstrained bargaining game with symmetric bargaining powers. In order to model such a game, it is habitual to use the Nash bargaining model (Nash (1950)), which seeks to find the t that maximises the Nash product, $N(t) = \pi(t)R(t)$.¹⁸ In this section we look at this option.

In order to calculate the Nash product, it is useful to firstly re-write the user's profit function. Starting from (1), we can substitute in the optimal

¹⁷For example, this could be the case when the copyright holder is a copyright collective, established in the interests of the efficiency gains from saving on transactions costs. The regulators may allow the collective to form, but may be worried about the ensuing monopoly power that is created. One way around this problem is to attempt to regulate the fee that the collective charges, such that the fee would represent a fair bargaining outcome.

¹⁸Of course the Nash bargaining model would really maximise the product of the difference between each utility and the corresponding disagreement utility. But for the case at hand, disagreement implies no deal at all, in which case both parties earn 0.

output and simplify to get

$$\pi(t) = \left(\frac{1}{4b}\right) \left(\frac{((1-t)-c)^2}{(1-t)}\right) \quad (12)$$

Deriving this, we find that¹⁹

$$\pi'(t) = -\frac{R(t)}{t}$$

Thus, the derivative of the Nash product is

$$\begin{aligned} N'(t) &= \pi'(t)R(t) + \pi(t)R'(t) \\ &= -\frac{R(t)^2}{t} + \pi(t)R'(t) \end{aligned} \quad (13)$$

Notice that, if the Nash product is concave, then its maximum occurs at t^N such that

$$\frac{R(t^N)^2}{t^N} = \pi(t^N)R'(t^N) > 0$$

This implies that it must be that $R'(t^N) > 0$, and since $R(t)$ is concave in t , and it is maximised at t^* (which is where $R'(t^*) = 0$) it holds that $t^N < t^*$. So, assuming that the Nash product has a maximum at a concave point, the tax rate implied is less than the optimal tax rate for the copyright holder. That is, (as expected) the Nash bargaining outcome is less favourable to the copyright holder, and more favourable to the user, than is the tax rate t^* . In essence, the Nash solution removes the copyright holder's monopoly power when the tax rate is set.

We are also able to prove the following interesting result:

Theorem 6. *At the Nash bargaining solution, the ratio of copyright holder income to user profit is equal to the elasticity of the copyright holder's income to the tax rate.*

Proof. The theorem can be easily proved by simple algebra on the first-order condition, $\frac{R(t^N)^2}{t^N} = \pi(t^N)R'(t^N)$. Dividing both sides of this equation by

¹⁹The working is in section 6 of the appendix.

$R(t^N)\pi(t^N)$ and multiplying both sides by t^N , gives $\frac{R(t^N)}{\pi(t^N)} = \frac{t^N R'(t^N)}{R(t^N)} = \varepsilon^R(t^N)$, where ε^R is the elasticity of the copyright holder's revenue to the tax rate. \square

Theorem 7. *The Nash bargaining solution tax rate equates the absolute values of the elasticities of each players' payoff with respect to the tax rate; $-\varepsilon^\pi(t^N) = \varepsilon^R(t^N)$.*

Proof. The proof is immediate from theorems 5 and 6. \square

It turns out that the Nash product is not everywhere concave, but it is concave around its maximum. This feature can be established by looking at the second derivative, but doing this is overly complex. Rather, we can look at the graph of the Nash product, and see its features there.

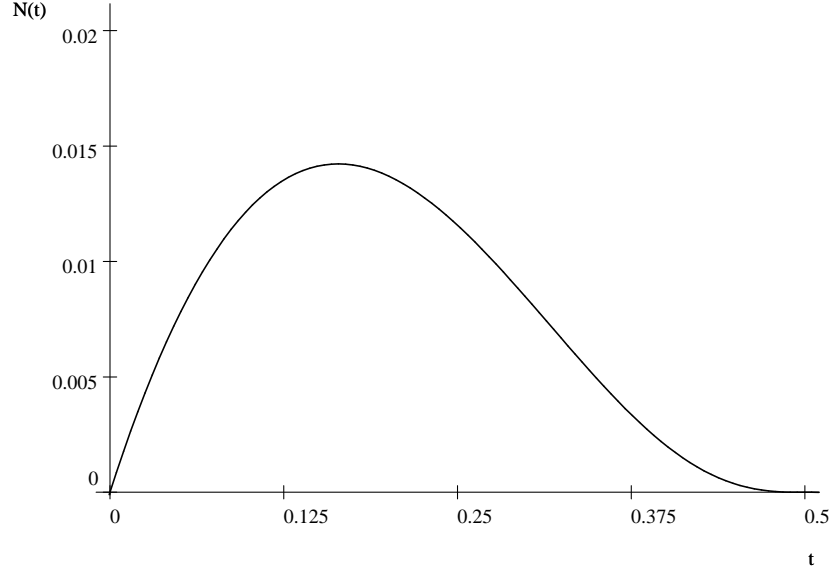
Using (12) together with (5), the Nash product is

$$\begin{aligned} N(t) &= \left(\frac{1}{4b}\right) \left(\frac{((1-t)-c)^2}{(1-t)}\right) \left(\frac{t((1-t)^2-c^2)}{4b(1-t)^2}\right) \\ &= \left(\frac{1}{4b}\right)^2 \left(\frac{t((1-t)-c)^2((1-t)^2-c^2)}{(1-t)^3}\right) \end{aligned}$$

Since $(1-t)^2 - c^2 = ((1-t)-c)((1-t)+c)$, we can write the Nash product as

$$\begin{aligned} N(t) &= \left(\frac{1}{4b}\right)^2 \left(\frac{t((1-t)-c)^2((1-t)-c)((1-t)+c)}{(1-t)^3}\right) \\ &= \left(\frac{1}{4b}\right)^2 \left(\frac{((1-t)-c)}{1-t}\right)^3 t((1-t)+c) \end{aligned} \quad (14)$$

Again, this is a rather complex looking equation, although it is certainly able to be analysed. The graph of the equation for $b = \frac{1}{4}$ and $c = \frac{1}{2}$ is displayed in Figure 4:

Figure 4: The graph of $N(t)$

Clearly, while not concave everywhere (specifically, it is not concave for values of t close to $\frac{1}{2}$), this graph has a local maximum at a strictly concave section of the graph. Indeed, $N(t)$ goes negative for values of t greater than $\frac{1}{2}$ (not shown in Figure 4) thus the maximum that is visible in the graph is the global optimum for the Nash product.

In order to locate the maximum of the Nash product, we need to find the t^N that solves $N'(t^N) = 0$. The first derivative of (14) can be expressed as²⁰

$$N'(t) = g(t) ((1 - t - c)(1 - 2t + c)(1 - t) - 3ct(1 - t + c)) \quad (15)$$

where $g(t) \equiv \left(\frac{1}{4b}\right)^2 \left(\frac{1-t-c}{1-t}\right)^2 \left(\frac{1}{1-t}\right)^2$. Since $g(t)$ is positive and finite, the first derivative is zero at

$$n(t, c) \equiv (1 - t - c)(1 - 2t + c)(1 - t) - 3ct(1 - t + c) = 0$$

²⁰The working is in section 7 of the appendix.

The graph of $n(t, c)$, for $c = \frac{1}{2}$ is drawn in Figure 5. The point at which it crosses the axis, i.e. the tax rate that maximises the Nash product, is $t^N = 0.16415$.

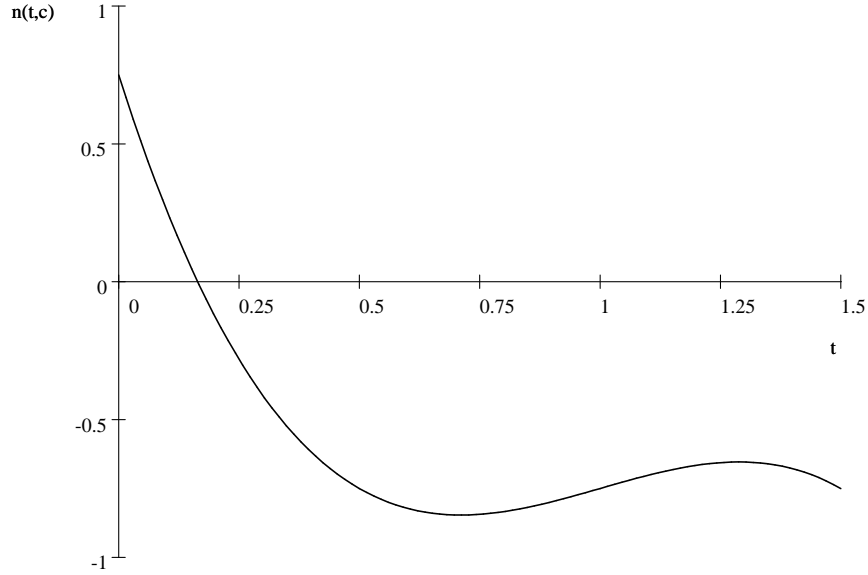


Figure 5: The shape of $n(t, c)$

Unfortunately, this time the algebraic solution to $N'(t^N) = 0$ proves to be extremely complex, and thus not particularly useful. Concretely, it turns out that the solution is given by (again, using the package DERIVE):

$$t^N(c) = \frac{1}{3}c + \sqrt[3]{k(c)} - \frac{\frac{2}{9}c^2 - \frac{2}{9}c - \frac{1}{36}}{\sqrt[3]{k(c)}} + \frac{5}{6}$$

where

$$k(c) \equiv \sqrt{\frac{1}{216}c^2 + \frac{13}{216}c^3 + \frac{19}{54}c^4 + \frac{1}{9}c^5 + \frac{1}{36}c^6 - \frac{5}{9}c^2 - \frac{7}{54}c^3 - \frac{1}{18}c - \frac{1}{216}}$$

The tax rate $t^N(c)$ is graphed in Figure 6. It is strictly decreasing and convex.

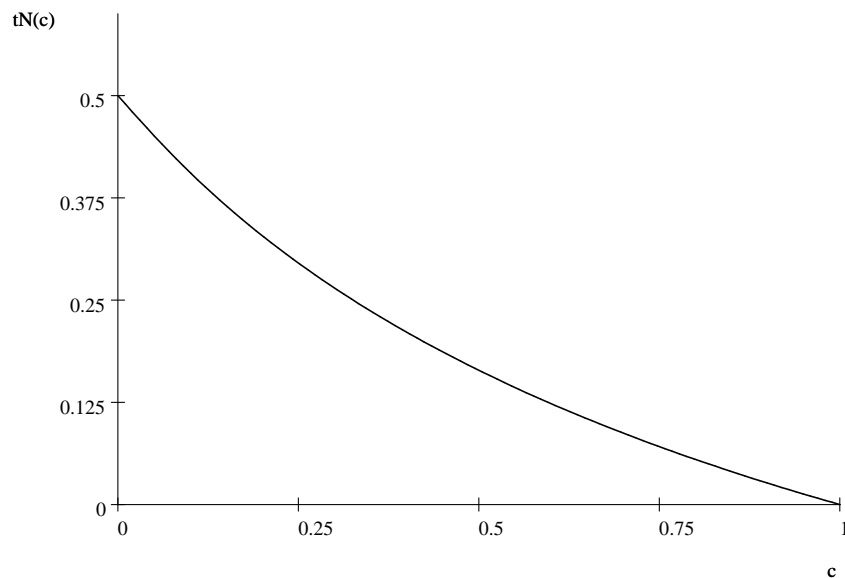


Figure 6: The Nash bargaining tax rate, t^N , as a function of c

The complexity of the algebraic form of $t^N(c)$ implies that it does not lend itself well to regulators. However, we can find a lower bound on t^N that does have a reasonably simple expression. To start with, it is worthwhile wondering why we might be interested in a lower bound rather than an upper bound. After all, from Figure 6, a reasonable upper bound is very easy to spot. We would just have to use $t^u = \frac{1-c}{2}$, which is the straight line joining the two end-points of the graph in Figure 6. This bound will always overstate the Nash bargaining solution tax rate (although admittedly not by much), and thus is more generous to the copyright holder and less generous to the user than is the Nash bargaining rate. If we recall that what we are looking for is a solution to the problem that is envisaged when the copyright holder has excessive monopoly power, and we are taking the Nash bargaining solution to be a way in which that market power is removed. Taking an upper bound on the Nash bargaining solution does not remove all of the market power, and the user may still complain about the dominant

position of the copyright holder. In order that this does not happen, we shall be much more interested in a lower bound on the Nash bargaining tax rate.

Notice from Figure 5 that $n(t, c)$ is convex to the left of the point t^N . Given that, the curve $n(t, c)$ lies above the tangent to this curve at $t = 0$. This implies that the value of t^N , the point at which $n(t, c)$ cuts the horizontal axis, must always be greater than (i.e. must always lie to the right of) the point at which the tangent line to $n(t, c)$ at $t = 0$ cuts the horizontal axis. Lets call that point t^{Ne} , since it is a value that estimates, from below, t^N .

In order to find an expression for t^{Ne} , we need to find the equation of the supporting tangent line of $n(t, c)$ at the point $t = 0$, and then find the point at which this tangent line cuts the horizontal axis. The derivative of $n(t, c)$ is

$$\frac{\partial n(t, c)}{\partial t} = (1 - t)(6t - 4) + c(4t - 2) - 2c^2$$

At $t = 0$, this is

$$\left. \frac{\partial n(t, c)}{\partial t} \right|_{t=0} = -4 - 2c - 2c^2$$

Also, note that $n(0, c) = (1 - c)(1 + c) = 1 - c^2$. So, the equation of the tangent line in question is

$$(1 - c^2) - (4 + 2c + 2c^2)t$$

and so the point at which the tangent line cuts the horizontal axis is given by

$$\begin{aligned} t^{Ne} &= \frac{1 - c^2}{4 + 2c + 2c^2} \\ &= \left(\frac{1}{2}\right) \left(\frac{1 - c^2}{2 + c + c^2}\right) \end{aligned} \quad (16)$$

For example, with $c = \frac{1}{2}$, for which $t^N = 0.16415$, we get $t^{Ne} = 0.13636$. In Figure 7, the straight line shows the tangent to $n(t, c)$ at $t = 0$, and the

point at which that line cuts the axis is t^{Ne} , which clearly lies to the left of t^N .

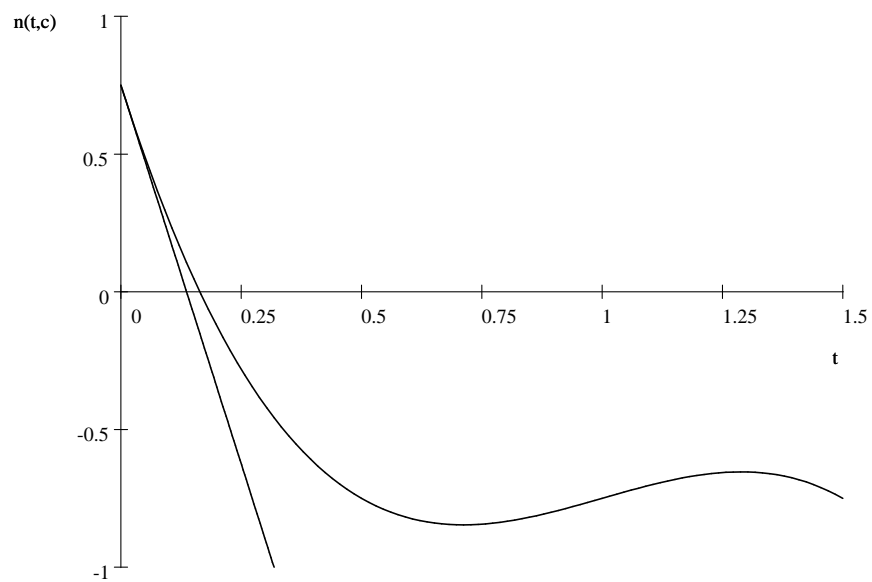


Figure 7: The relationship between t^N and t^{Ne}

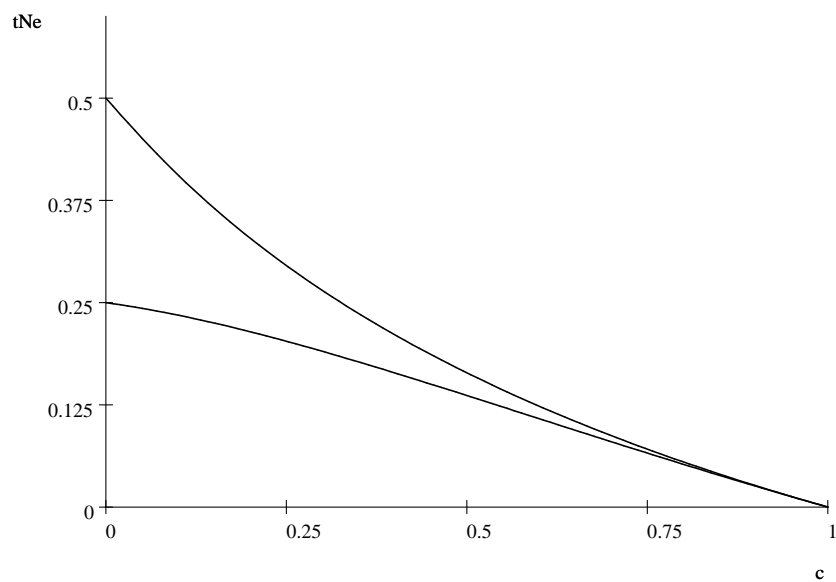


Figure 8: The shape of $t^{Ne}(c)$ in comparison with $t^N(c)$

The graph of the lower bound on the Nash tax rate, as a function of c , is given in Figure 8 (the lower curve), along with the Nash rate (the higher curve). The graph of $t^{Ne}(c)$ is a strictly decreasing and concave function.

From Figure 8, we can see that $t^{Ne}(c)$ better approximates $t^N(c)$ the greater is c . It is also evident that, given the shape of $t^N(c)$, other approximations are surely feasible, so $t^{Ne}(c)$ should not be thought of as the only approximating lower bound function. For example, it happens that the slope²¹ of the function $t^N(c)$ is -1 at the point $c = 0$, and the slope is $-\frac{1}{4}$ at the point $c = 1$. Thus, the function $t^N(c)$ lies everywhere above the two straight lines

$$\begin{aligned} t &= \frac{1}{2} - c \\ t &= \frac{1}{4}(1 - c) \end{aligned}$$

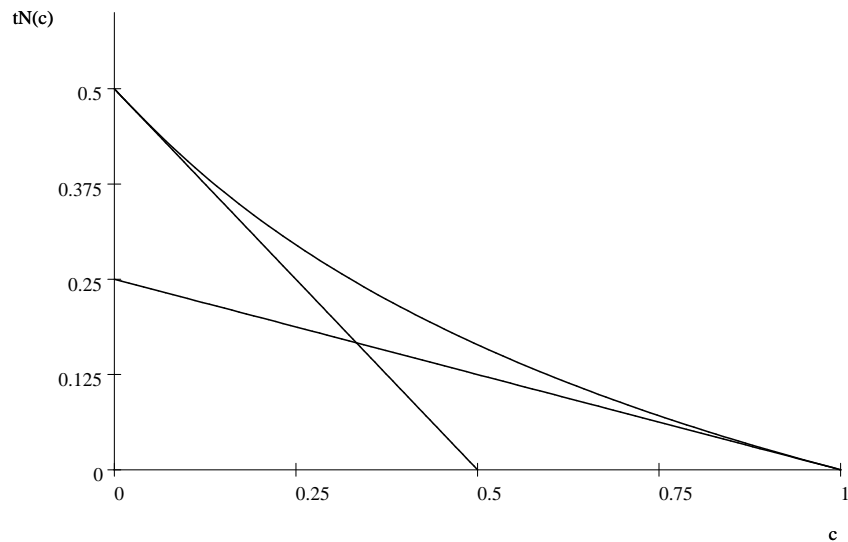


Figure 9: The Nash bargaining tax rate, t^N , with two of its supporting tangents

²¹Of course, evaluating the slope of $t^N(c)$ is very complex. I used Matlab to evaluate the derivative, and to calculate its value at the endpoints.

The two lines, $t = \frac{1}{2} - c$ and $t = \frac{1}{4}(1 - c)$, along with $t^N(c)$ are shown in Figure 9. The flatter of the two supporting tangent lines in Figure 9 alone actually provides a very easy to calculate approximation to $t^{Ne}(c)$. Since $t^{Ne}(c)$ is concave, it lies everywhere above the straight line through its endpoints. This straight line is $t = \frac{1}{4}(1 - c)$. Thus, $\frac{1}{4}(1 - c) \leq t^{Ne}(c)$ for all c . Since $t^{Ne}(c)$ is not very concave, the approximation should be reasonably good, and clearly the equation $\frac{1}{4}(1 - c)$ is extremely simple. For the case of $c = \frac{1}{2}$, for which $t^N = 0.16415$, and $t^{Ne} = 0.13636$, we get $\frac{1}{4}(1 - c) = 0.125$.

Taken together, we can also use the two supporting tangent lines to get an approximation to the Nash bargaining tax rate that is accurate for either large or small values of c , and less accurate for intermediate values. The two supporting tangent lines intersect at $c = \frac{1}{3}$, at which point $t = \frac{1}{6}$. Given this, consider the upper envelope of the two straight lines, that is, the piecewise function

$$t^E(c) = \begin{cases} \frac{1}{2} - c & \text{for } 0 \leq c \leq \frac{1}{3} \\ \frac{1}{4}(1 - c) & \text{for } \frac{1}{3} \leq c \leq 1 \end{cases}$$

Clearly, $t^E(c)$ also provides a lower bound on the Nash bargaining tax rate. This rate is a better approximation than is $t^{Ne}(c)$ to the Nash rate for low values of c , but a slightly worse approximation for high values of c . Of course, we could also look for the upper envelope of the two functions $t = \frac{1}{2} - c$ and $t^{Ne}(c)$, which would give us a better approximation than just $t^{Ne}(c)$ for low values of c and the same approximation for high values of c . Any number of other options can be devised as well.²² However, since it seems a little

²²Indeed, any supporting tangent line to the function $t^N(c)$ must lie everywhere below the function. There are, of course, infinite such supporting tangent lines. One particularly simple one to envisage would be that with slope equal to $-\frac{1}{2}$. Geometrically, in Figure 9 draw the straight line joining the two endpoints of $t^N(c)$, and then move it downwards in a parallel manner until it is just tangent to the curve $t^N(c)$. That tangency must happen at a strictly interior point. The equation thus obtained would be of the form $t^e = z - \frac{c}{2}$. This approximation would be very good for intermediate values of c and worse for either very large or very small values of c . However, the number z , which is strictly less than $\frac{1}{2}$, may have a rather complex form.

arbitrary to choose one option over any other, in what follows I shall simply stick with $t^{Ne}(c)$ as the lower bound approximation to the Nash solution.

We can consider the way in which the total surplus is shared when the lower bound to the Nash solution is used. To do that, we only need to substitute (16) into (11). This gives us the equation²³

$$\frac{\pi(t^{Ne})}{R(t^{Ne})} = \left(\frac{3 + 4c + 5c^2}{1 - c^2} \right) \left(\frac{3 + 3c^2 - 2c^3}{3 + 8c + 7c^2 + 2c^3} \right)$$

Again, a rather formidable looking expression, but one that can be easily graphed. The graph of the surplus sharing regime under the lower bound tax rate is given in Figure 10.

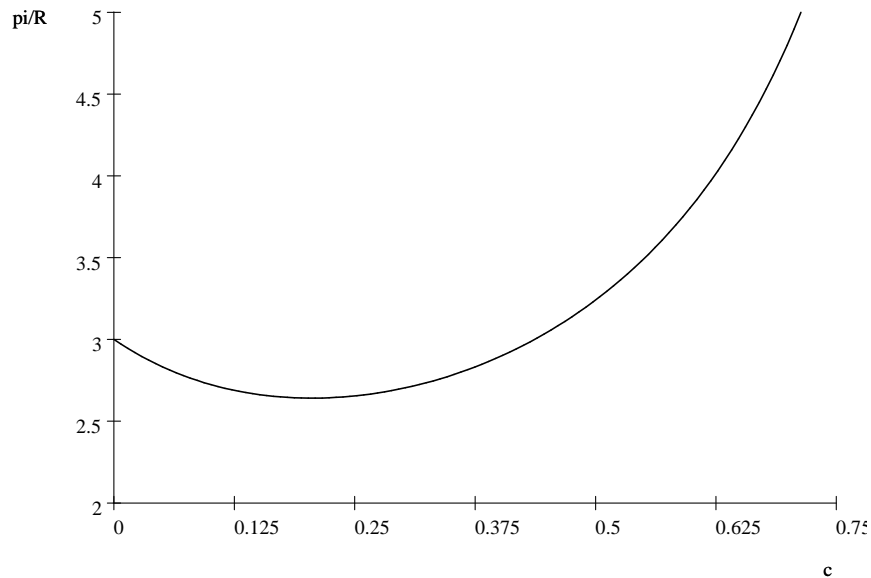


Figure 10: The shape of $\frac{\pi(t^{Ne})}{R(t^{Ne})}$ for small values of c

The curve is everywhere convex, valued at 3 when $c = 0$, goes infinite as $c \rightarrow \infty$, and has a minimum at c approximately equal to 0.2055. At the minimum of the curve, the value of $\frac{\pi}{R}$ is clearly greater than 2.5 (actually, its minimum value is about 2.6414), and so for this tax rate function, the

²³See section 8 of the appendix for the working.

user always gets the lion's share of the total surplus (the user's profit is at least two and a half times greater than the copyright holder's income for all possible values of c).

6. FIXED COSTS

One concern with the modelling above is the assumption that the user produces with no fixed costs. In reality, this assumption does not alter the way in which either the copyright holder's optimal revenue tax, or the Nash bargaining revenue tax are calculated. Of course, it also does not affect the calculation of the user's optimal output, unless it serves to set optimal output to 0. The optimal revenue tax is simply found where the copyright holder's income is maximised, which is independent of any fixed costs in the user's production process. In the Nash model, in reality the Nash product is

$$N(t) = (R(t) - \bar{R}) (\pi(t) - \bar{\pi})$$

where \bar{R} and $\bar{\pi}$ are, respectively, the earnings of the copyright holder and the user when no deal is struck. When fixed costs are 0, since no deal means no surplus to share, we get $\bar{R} = \bar{\pi} = 0$. When there are fixed costs of F , we still have $\bar{R} = 0$, but now we have $\pi_F(t) = \pi(t) - F$, and $\bar{\pi} = -F$. Therefore, we would have

$$\begin{aligned} N(t) &= (R(t) - \bar{R}) (\pi_F(t) - \bar{\pi}) \\ &= R(t)(\pi(t) - F + F) \\ &= R(t)\pi(t) \end{aligned}$$

The *only* effect of including fixed costs for the user is that the upper limit revenue tax rate that can be charged is altered, and may end up being a binding constraint upon the problem.

Specifically, since we know that $[(1 - \bar{t})(1 - bx^*(\bar{t})) - c]x^*(\bar{t})$ is decreasing in the tax rate, when the user faces fixed costs of F , the tax rate must be set such that

$$\pi_F(t) = [(1 - t)(1 - bx^*(t)) - c]x^*(t) - F \geq 0$$

Using (12), the requirement is that

$$\left(\frac{1}{4b}\right) \left(\frac{(1 - t - c)^2}{(1 - t)}\right) \geq F$$

that is

$$(1 - t - c)^2 - (1 - t)4bF \geq 0$$

Consider the function $h(t) \equiv (1 - t - c)^2 - (1 - t)4bF$. Its slopes are

$$h'(t) = -2(1 - t - c) + 4bF$$

$$h''(t) = 2 > 0$$

$$\frac{\partial h}{\partial F} = -(1 - t)4b < 0$$

Since its second derivative is positive, $h(t)$ is a convex function. Since the derivative in F is negative, an increase in F shifts the function downwards.

Let's start with the case of $F = 0$. In that case, $h(t)$ reaches its minimum exactly at the horizontal axis. The graph in question is shown in Figure 11, in which $c = \frac{1}{2}$. The higher graph is that corresponding to $F = 0$. At the point at which that graph touches the horizontal axis, we have $t = 1 - c$. We are only concerned with the decreasing part of the function, since we know that we must always restrict $t \leq 1 - c$ in order that output be non-negative. In the case of $F = 0$, the entire negatively sloped part of the function is valued positive, and so (as we already saw above), in this case the only restriction on t is that it cannot go above $1 - c$.

Now, as F goes positive, the function moves downwards. This will generate two roots of the function, one above $1 - c$ (where the function will have positive slope) and one below $1 - c$ (where the function will have negative

slope). Such a case is drawn as the lower graph in Figure 11, where it is assumed that $c = \frac{1}{2}$ and $bF = \frac{1}{40}$.

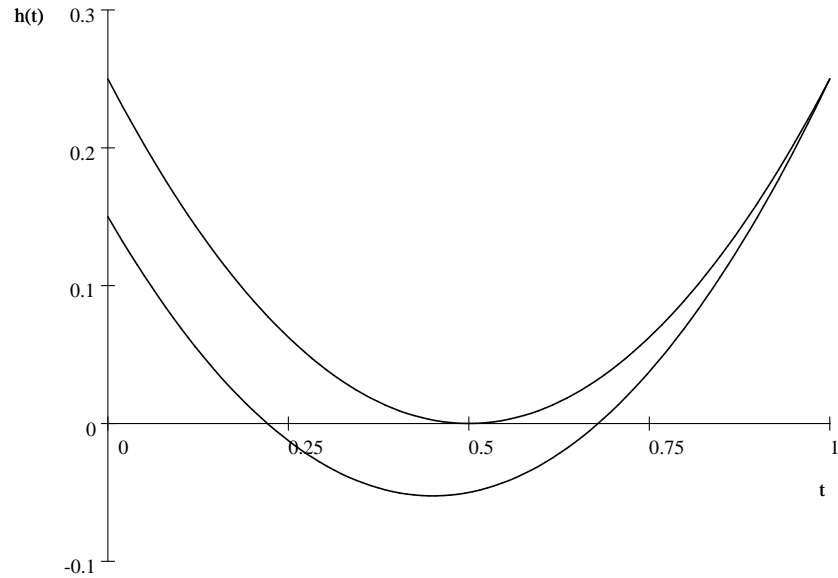


Figure 11: The shape of $h(t)$ for $F = 0$ (higher graph) and for $F > 0$ (lower graph).

Since the function $h(t)$ is negative between the lower root and the value $1 - c$, we know that we can only consider as valid tax rates those that are no greater than the lower root. Using the quadratic formula, it can be shown that the lower root is at

$$t_0(c, F) = (1 - c - 2bF) - 2\sqrt{bF}\sqrt{bF + c} \quad (17)$$

Now, note that if the revenue tax were to be set at 0, the user receives the copyright work free of charge. It is not conceivable that this would result in a negative overall profit. If it did, then really there is no reasonable business proposal in the first place. Thus, at $t = 0$, it is entirely reasonable that the user earns a strictly positive profit. In terms of our graphs, this converts to

the condition that $h(0) > 0$, that is

$$1 - 4bF - 2c - c^2 > 0$$

or

$$\left(\frac{1}{4b}\right) (1 - 2c - c^2) > F$$

Note that this condition implies that $t_0(c, F) > 0$, i.e. there is always something that can be shared.

The graph of $t_0(c, F)$, for $bF = \frac{1}{40}$ is displayed in Figure 12. It is a strictly decreasing and (slightly) convex function.

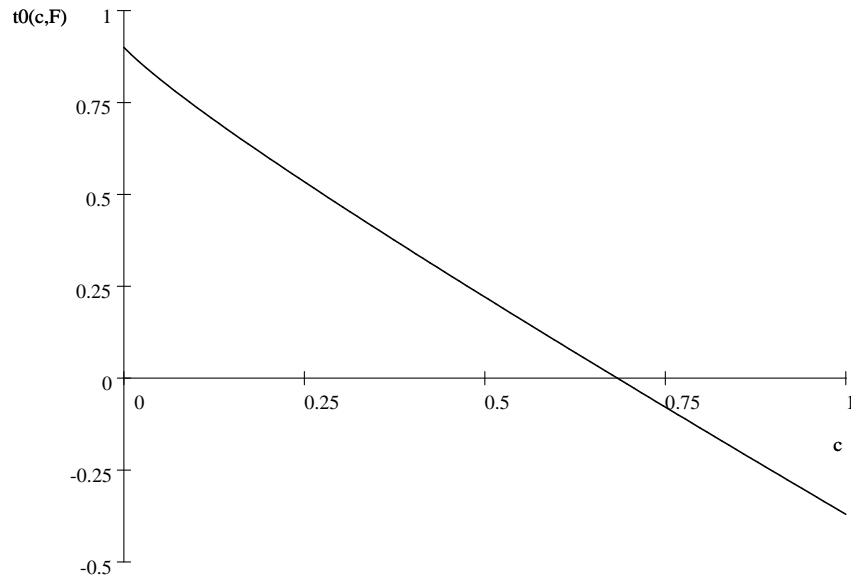


Figure 12: The limit tax rate as a function of c , with positive fixed costs

Notice that, for $bF = \frac{1}{40}$, the limit tax rate is only positive for low enough values of c . In fact, with this example, the limit tax rate goes to 0 at $c = 0.68377$. All this is really saying is that when the marginal cost is high enough and a fixed cost is present, the user's business becomes unprofitable, since even if the copyrighted input were supplied free of charge, the user still makes a non-positive profit.

As F goes up, the graph of the limit tax rate moves downwards. This clearly implies that, if it is intended that the lower bound on the Nash tax rate should be used, there is a maximum admissible level of fixed costs that can be present. Specifically, recall that the graph of $t^{Ne}(c)$ is strictly negatively sloped, and $t^{Ne}(0) = \frac{1}{4}$. Thus for all values of c we must have $t^{Ne}(c) < \frac{1}{4}$. It also happens that $t^{Ne}(c)$ is a flatter function than is $t_0(c, F)$.²⁴ Thus, if this lower bound tax rate is ever to be feasible at any values of c , we require that the limit tax rate at $c = 0$ be no smaller than $\frac{1}{4}$, otherwise the limit tax rate will be smaller than the lower bound rate for all c . In such a case, even the lower bound on the Nash solution tax rate would be too high for the user to want to participate in the venture, since participation would result in negative profits. The restriction then is that $t_0(0, F) = (1 - 2bF) - 2\sqrt{bF}\sqrt{bF} = 1 - 4bF \geq \frac{1}{4}$. With a very minimal amount of effort, this restriction can be expressed as

$$bF \leq \frac{3}{16} \quad (18)$$

Of course, even if restriction (18) is satisfied, there is no *guarantee* that the lower bound on the Nash solution tax rate is feasible. It depends upon the value of the marginal cost c . The restriction (18) is thus a necessary, but not sufficient, condition for the lower bound on the Nash tax rate to be feasible.

Finally, in Figure 13 we graph the limit tax rate (the curve that is second highest at the vertical axis) together with the optimal tax rate (the curve that cuts the vertical axis the highest) and the lower bound on the Nash solution tax rate (the curve that cuts the vertical axis the lowest). While not

²⁴The slope of $t^{Ne}(c)$ can be calculated as $-\left(\frac{1}{c+c^2+2}\right)\left(c + \frac{(2c+1)(1-c^2)}{c+c^2+2}\right)$. We also know that $t^{Ne}(c)$ is a decreasing concave function, thus it takes its smallest slope (i.e. its steepest part) at $c = 1$, where its slope is $-\frac{1}{4}$. Thus, the slope of $t^{Ne}(c)$ is everywhere greater than $-\frac{1}{4}$. On the other hand, the slope of $t_0(c, F)$ in c turns out to be $-1 - \sqrt{\frac{bF}{bF+c}}$ which is clearly less than -1 for all c . Thus, for all c the function $t_0(c, F)$ takes a smaller negative slope (i.e. is steeper) than the function $t^{Ne}(c)$.

really visible in the graph (where it appears that the limit tax rate forms a tangent with the Nash solution tax rate), for the parameters chosen, it turns out that the Nash solution tax rate is actually lower than the limit rate, but only for a very small range of value of c . Concretely, it turns out that between about $c = 0.1162$ and $c = 0.2$, the Nash solution tax rate dips below the limit rate (although only very very slightly). Thus (at least for the parameter values used here), the fixed cost makes the Nash solution tax rate impossible for all but a very small range of values of c . Since the limit tax function shifts downwards with an increase in F , we can easily see that there will be values of F (not significantly greater than the $\frac{1}{10}$ that is assumed in the graph) for which there would be no values of c for which the Nash solution tax rate is feasible.

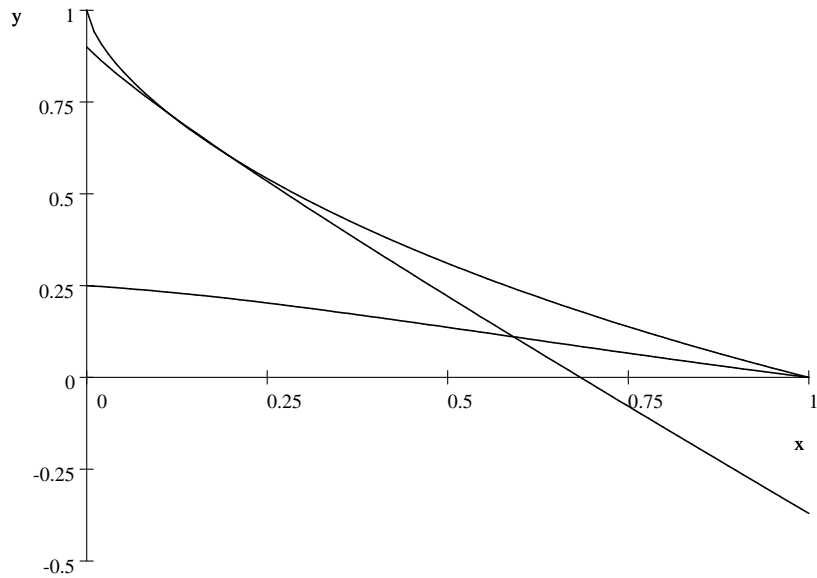


Figure 13: Comparison between the limit tax rate, the Nash solution tax rate, and the lower bound Nash tax rate

However, our lower bound on the Nash tax rate is clearly lower than the limit tax for all but quite large levels of c . Thus the lower bound on the Nash

tax rate is feasible for a large set of values of marginal cost, and therefore it can be used as a tax on revenues without fear that the copyright user will be forced into negative profits, at least over that range of values of c for which the limit is above the lower bound on the Nash tax rate. Concretely, with the parameters used in this example, the lower bound on the Nash tax rate is below the limit tax rate for all values of c less than 0.59116.

7. REGULATORY POLICY RECOMMENDATIONS

While simplified with respect to the real world, the above model allows us to grasp some initial insights into the contracts under which access is granted to copyright works, in which the contracts are stipulated to be a revenue sharing arrangement between the user and the copyright holder. The model looks at both optimal sharing arrangements from the perspective of the copyright holder,²⁵ and the outcome of an evenly structured Nash bargaining game (along with an interesting lower bound on the tax rate that would emerge from such a bargaining game). The model only requires knowledge of a few concrete variables in order for it to provide guidance as to the optimal tax rate, and of course as to the lower bound on the bargaining tax rate. Specifically, we need to know the value of c , the marginal cost as a fraction of the vertical intercept of the demand function.²⁶ If there is a fixed cost present in the user's production process, then F would also have to be known, since the fixed cost alters the set of feasible tax rates that can be used, as would the slope parameter of the demand curve.

The most interesting rate from the perspective of regulatory policy is the lower bound on the Nash bargaining rate, since regulators are typically charged with finding fair and equitable solutions to these sorts of problems.

²⁵Of course, an optimal arrangement from the perspective of the user is to simply set the tax rate to 0.

²⁶If, in a real-world setting, the demand function and/or the cost function were estimated to be non-linear, it would be necessary to simply extract the linear approximation to each curve in order to apply the model.

The Nash bargaining model posits an unrestricted bargaining process, with equal bargaining powers. Thus the rate that emerges from the Nash bargaining game is fair and equitable to all parties. However, the Nash solution rate is too complex, and that is why the lower bound might be of interest to regulators. When compared to the Nash solution, the lower bound that we have established for the Nash rate is more favourable to the user than to the copyright holder, and thus it is very hard to argue that this lower bound does not ameliorate the monopoly power that the copyright holder may have when the copyright holder is in fact a collective. It also happens that the lower bound on the Nash rate is likely to be feasible when there is a fixed cost present in the problem, at least for a reasonably large set of parameters. Thus, the lower bound on the tax rate may go some distance in solving the problem that is often faced by regulators when they attempt to set a fair revenue tariff.

Given that there are likely to be some fixed costs in the user's production function, my suggestion for a regulated revenue tax is

$$t_r = \min \left\{ \left(\frac{1}{2} \right) \left(\frac{1 - c^2}{2 + c + c^2} \right), (1 - c - 2bF) - 2\sqrt{bF}\sqrt{bF + c} \right\} \quad (19)$$

We can use this to get an idea about how well regulators in the real world have managed to set a fair rate. In order to do this, I shall posit a simple example, based on the broadcast music radio business. In the music radio business, where a regulated revenue tax is habitually used, this tax rate is normally held under 5%, although there are cases in which a rate as high as 10% is levied on some radio stations.²⁷

Consider the traditional radio broadcasting business. The radio station requires music in order to function, and the music is supplied as an entire repertory by the copyright holders acting together as a copyright collective.

²⁷This is the case, for example, in Ireland and India, for radio stations that are particularly high users of music as an input.

The good produced, x , is audience which is then sold to advertisers. Since advertisers will have decreasing marginal willingness to pay for an additional unit of audience, it is certainly reasonable that the demand curve for advertising faced by a radio station is decreasing in the audience size, just as has been assumed in this paper. Thus, it is reasonable to assume that there is indeed some price at which demand would go to 0 (at least, in a linear approximation to the demand curve).

Consider the cost situation of a typical radio station. The radio business is largely run according to a cost model that is independent of the audience reached.²⁸ The main costs are to obtain frequency, purchasing and maintaining broadcasting equipment, office space, wages of disc-jockies, program directors and executives, etc.²⁹ In fact the only cost that would be associated with altering the audience would seem to be those costs incurred in marketing.

Take a radio station operating in a medium sized city. Assume that the demand curve for advertising is given by³⁰

$$p = 1 - \frac{x}{8,000,000}$$

Say that outside of any payment for the music input (i.e. for now we set $t = 0$), the station has \$1.5 million in total costs. Let's say frequency costs, equipment, office space, wages etc. (i.e. costs that are independent of audience) all add up to \$1.3 million, leaving \$200,000 per year for marketing (the only variable cost). We know that the station will set output (i.e.

²⁸There is a difference here with internet broadcast radio. When more audience is to be reached, greater bandwidth is required, and so there is a strong element of variable cost in the internet radio business.

²⁹Of course the radio must also pay collecting organisations for the right to broadcast music. However, since this is exactly the fee that we are interested in calculating here, we shall deal with it separately.

³⁰Here, x is the number of listeners of the station, averaged over all broadcast minutes.

audience)³¹ according to (3), which with $t = 0$ is

$$\frac{1 - c}{2b} = \frac{8,000,000(1 - c)}{2} = 4,000,000(1 - c)$$

The average variable cost (per unit audience) is thus equal to

$$\frac{200,000}{4,000,000(1 - c)} = 0.05(1 - c)$$

If the cost function is indeed linear, then average variable cost is equal to marginal cost, so

$$c = 0.05(1 - c)$$

from which $c = 0.0476$.

Now, we also know that the profit in general is given by

$$\pi = \left(\frac{1}{4b}\right) \left(\frac{(1 - t - c)^2}{(1 - t)}\right) - F$$

Substituting in the numbers that we have ($t = 0$, $b = \frac{1}{8,000,000}$, $c = 0.0476$, and $F = 1,300,000$), we get

$$\begin{aligned} \pi &= \frac{8,000,000(1 - 0.0476)^2}{4} - 1,300,000 \\ &= 2,000,000(1 - 0.0476)^2 - 1,300,000 \\ &= 514,131.52 \end{aligned}$$

The lower bound on the Nash tax rate is

$$\left(\frac{1}{2}\right) \left(\frac{1 - c^2}{2 + c + c^2}\right) = \left(\frac{1}{2}\right) \left(\frac{1 - 0.0476^2}{2 + 0.0476 + 0.0476^2}\right) = 0.24337$$

And, since $bF = \frac{1,300,000}{8,000,000} = 0.1625$ the limit tax rate is

³¹Of course, in the real world of music radio, the station may not be able to easily pick and choose its audience level. The example is only meant to be illustrative. If the audience achieved is not the optimal level, then the station will simply earn less revenue and profit than would be feasible in the optimum. This will typically mean that the restriction on the maximum revenue sharing tax is reached earlier. The example can be easily modified to fit the case of sub-optimal audience levels.

$$\begin{aligned}
(1 - c - 2bF) - 2\sqrt{bF}\sqrt{bF + c} &= (1 - 0.0476 - 0.325) \\
&\quad - 2\sqrt{0.1625}\sqrt{0.1625 + 0.0476} \\
&= 0.25785
\end{aligned}$$

Since the limit rate is greater than the Nash rate, the Nash rate is a feasible tax (i.e. it leaves the user with strictly positive profit). Thus, with this example, the regulator should set the revenue tax rate at about 24.3%. At that tax rate, the user still earns strictly positive profit, and the rate is, by definition, fair to both parties.³²

At the tax rate of 0.243, using (12) and (5) we can calculate that the total profit retained by the user is \$29,586.16, and the copyright collective earns income of \$484,078.42. It is, perhaps, instructive to note that, when the revenue tax is installed, the total surplus is $29,586.16 + 484,078.42 = 513,664.58$, which is smaller than the total surplus when the tax is set at 0 (which is 514,131.52). Thus, it would be better for the copyright collective to set a tax rate of 0, and to charge a fixed fee of $514,131.52 - 29,586.16 = 484,545.36$. If this could be done, the user would be indifferent to the fixed fee situation and the regulated revenue tax, but the copyright collective would be better off. This example serves to show that although in some situations the regulator makes a revenue tax the compulsory compensation mechanism, this may be inefficient.

8. CONCLUSIONS

In this paper I have analysed a model in which an copyright holder supplies an essential input to a user, and in exchange for the input the copyright

³²Since this is only a fictitious example, we should not read too much into the large difference between the revenue tax rate that the model gives (24.3%) and the real-world rates of closer to 5%. Until real-world data can be brought to the model to calibrate it, we cannot know how the 5% standard compares to a fair negotiated rate.

holder is remunerated under a revenue sharing rule, that is, by a tax on the user's revenue. The analysis shows that, if there are no fixed costs in the user's production process, the optimal tax rate (from the point of view of the copyright holder) will always leave the user with positive earnings, but in most situations (those with a strictly positive marginal cost) will imply that the copyright holder earns more of the total surplus that is generated than does the user.

If the monopoly power that the copyright holder has when the remuneration system is negotiated is considered to be excessive, then one could look to the Nash bargaining model for guidance as to a fair revenue tax. The Nash bargaining model removes any monopoly power that the copyright holder has over the user when the revenue tax is set, and thus it constitutes a fair tariff. It turns out that while the Nash tax rate can always be calculated (once the parameters of the model are known), it is excessively complex. However, an interesting lower bound on the Nash tax rate can be found, and it has a relatively simple expression. Using the lower bound on the Nash tax rate, if the user's production process has no fixed costs, then contrary to when an optimal revenue tax is used, the user will always get a significantly greater share of the total surplus than would the copyright holder.

In the paper we have looked at both the simple case in which the user operates with no fixed costs, and the more complex case in which fixed costs are present. The presence of fixed costs does not alter the calculation of either the optimal revenue tax rate or the Nash tax rate (or the lower bound on the Nash tax rate), but it does imply that there is an upper bound on the tax rate that can be used. However, we have shown (by example) that while the upper bound might be a serious impediment to the use of an optimal

tax rate, it may leave plenty of scope for the use of the lower bound on the Nash tax rate.

It is worthwhile to point out that, if a regulator is interested in devising a revenue tax as remuneration for an essential input like the copyright to music played on music radio stations, it is of great aid that the formula for the tax be as simple and user-friendly as possible. The tax suggested in this paper seems to satisfy these requirements. In order to calculate the lower bound on the Nash tax rate, the regulator only requires to know the levels of marginal and fixed costs, and the vertical intercept of the demand curve for the producer's output. This vertical intercept could, of course, more realistically be found in a linear approximation to the demand curve.

The model in the present paper is only put forward as a first step, and it can be improved upon and more generally modified in several interesting ways.

The most obvious extension to the present model would be to calibrate it against some real-world data. While I do not have any hard data on hand concerning the operation of music radio generally, I conjecture that the lower bound on the Nash tax rate suggested in this paper is significantly greater than the tariffs that are levied in the real world as a result of regulatory practices in music radio. Since the lower bound is based upon a fair and unconstrained bargaining game, it removes any monopoly power held by the copyright holder, and so I conjecture that the regulated tariffs are generally too low. However, I stress that this is only conjecture, and the true relationship between the real-world regulated rates and the fair rate suggested here can only be known once some real-world data is inserted into the present model.

At a theoretical level, it is interesting to wonder what would happen if instead of supplying the input to a single user, the copyright holder were

to deal with many users? That is, what if the user actually operated in an oligopolistic market, rather than being a monopolist. So long as the copyright holder is obliged not to refuse any user (compulsory licensing), and is obliged not to discriminate among users in terms of price, the model might not change much from what is present in the current paper. Still, it would be interesting to find out.

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Appendix

In several places in the following proofs, we need to use the value of the market price of the user’s good, $1 - bx^*(t)$. Since we have

$$x^*(t) = \frac{(1-t) - c}{2b(1-t)}$$

it turns out that

$$\begin{aligned} 1 - bx^*(t) &= 1 - b \left(\frac{(1-t) - c}{2b(1-t)} \right) \\ &= 1 - \left(\frac{(1-t) - c}{2(1-t)} \right) \end{aligned}$$

Giving this a common denominator, we get

$$\begin{aligned} 1 - bx^*(t) &= \frac{2(1-t) - (1-t) + c}{2(1-t)} \\ &= \frac{(1-t) + c}{2(1-t)} \end{aligned} \tag{20}$$

1) In Theorem 1, we are interested in finding the tax rate that sets $(1 - \bar{t}_2)(1 - bx^*(\bar{t}_2)) - c = 0$. Substituting for the price from (20), we get

$$(1 - \bar{t}_2) \left(\frac{(1 - \bar{t}_2) + c}{2(1 - \bar{t}_2)} \right) = c$$

Cancelling the $(1 - \bar{t})$ that appears now in both the numerator and denominator of the left-hand-side gives

$$\frac{(1 - \bar{t}_2) + c}{2} = c$$

which simplifies directly to the equation given in the text.

2) To find the copyright holder's income function, firstly substitute (3) and (20) into (2);

$$\begin{aligned} R(t) &= t(1 - bx^*(t))x^*(t) \\ &= t \left(\frac{(1 - t) + c}{2(1 - t)} \right) \left(\frac{(1 - t) - c}{2b(1 - t)} \right) \end{aligned}$$

Combining the terms in both the numerator and denominator we get

$$\begin{aligned} R(t) &= \frac{t((1 - t) + c)((1 - t) - c)}{4b(1 - t)^2} \\ &= \frac{t((1 - t)^2 - c^2)}{4b(1 - t)^2} \end{aligned}$$

which is the equation given in the text.

3) The working necessary to find the derivatives of $R(t)$ is the following.

Firstly, write $R(t)$ as

$$\begin{aligned} R(t) &= \left(\frac{1}{4b} \right) \left(\frac{t((1 - t)^2 - c^2)}{(1 - t)^2} \right) \\ &= \left(\frac{1}{4b} \right) \left(t - c^2 \frac{t}{(1 - t)^2} \right) \end{aligned}$$

Then, using the quotient rule, we get

$$\begin{aligned} R'(t) &= \left(\frac{1}{4b}\right) \left(1 - c^2 \frac{(1-t)^2 + 2t(1-t)}{(1-t)^4}\right) \\ &= \left(\frac{1}{4b}\right) \left(1 - c^2 \frac{(1-t) + 2t}{(1-t)^3}\right) \\ &= \left(\frac{1}{4b}\right) \left(1 - \frac{(1+t)c^2}{(1-t)^3}\right) \end{aligned}$$

This is the equation given in the text.

To find the second derivative, again we use the quotient rule;

$$R''(t) = - \left(\frac{1}{4b}\right) \left(\frac{c^2(1-t)^3 + (1+t)c^2 3(1-t)^2}{(1-t)^6}\right)$$

Cancelling the $(1-t)^2$, we get

$$R''(t) = - \left(\frac{1}{4b}\right) \left(\frac{c^2(1-t) + (1+t)c^2 3}{(1-t)^4}\right)$$

Again, we only need to simplify the numerator. Taking out the common factor, c^2 , and expanding the numerator gives

$$R''(t) = - \left(\frac{c^2}{4b}\right) \left(\frac{4 + 2t}{(1-t)^4}\right)$$

Finally, dividing top and bottom by 2 gives the equation in the text

$$R''(t) = - \left(\frac{c^2}{2b}\right) \left(\frac{2 + t}{(1-t)^4}\right)$$

4) To express the user's profit as a function of the copyright holder's income, we firstly split the profit equation into three separate terms;

$$\begin{aligned} \pi(t) &= [(1-t)(1 - bx^*(t)) - c] x^*(t) \\ &= (1 - bx^*(t))x^*(t) - t(1 - bx^*(t))x^*(t) - x^*(t)c \end{aligned}$$

But, the second of those terms is equal to the copyright holder's income, and the first term is copyright holder income divided by t ;

$$\pi(t) = \frac{R(t)}{t} - R(t) - x^*(t)c$$

Joining the first two terms gives the equation in the text;

$$\pi(t) = R(t) \left(\frac{1-t}{t} \right) - x^*(t)c$$

5) In order to get the reduced form for the ratio of profit to copyright holder income, substituting the optimal output level into (10) to get

$$\frac{\pi(t)}{R(t)} = \left(\frac{1-t}{t} \right) - \frac{c}{t \left[1 - b \left(\frac{(1-t)-c}{2b(1-t)} \right) \right]}$$

The square bracket term is just the price, so substitute in from (20) to get

$$\begin{aligned} \frac{\pi(t)}{R(t)} &= \left(\frac{1-t}{t} \right) - \frac{c}{t \left(\frac{(1-t)+c}{2(1-t)} \right)} \\ &= \left(\frac{1-t}{t} \right) - \left(\frac{c}{(1-t)+c} \right) \left(\frac{2(1-t)}{t} \right) \end{aligned}$$

Now we see that there is a common factor of $\frac{1-t}{t}$, which we extract to get

$$\frac{\pi(t)}{R(t)} = \left(\frac{1-t}{t} \right) \left[1 - \frac{2c}{(1-t)+c} \right]$$

Finally, giving the square bracket term a common denominator and simplifying gives the equation in the text;

$$\begin{aligned} \frac{\pi(t)}{R(t)} &= \left(\frac{1-t}{t} \right) \left[\frac{(1-t)+c-2c}{(1-t)+c} \right] \\ &= \left(\frac{1-t}{t} \right) \left[\frac{(1-t)-c}{(1-t)+c} \right] \end{aligned}$$

6) The user's profit is

$$\pi(t) = \left(\frac{1}{4b} \right) \left(\frac{(1-t-c)^2}{1-t} \right)$$

Using the quotient rule, the derivative of this is

$$\pi'(t) = \left(\frac{1}{4b} \right) \left(\frac{-2(1-t-c)(1-t) + (1-t-c)^2}{(1-t)^2} \right)$$

Taking out the common factor in the second bracketed term, this is

$$\begin{aligned}
\pi'(t) &= \left(\frac{1}{4b}\right) \left(\frac{(1-t-c)(-2(1-t) + (1-t-c))}{(1-t)^2}\right) \\
&= \left(\frac{1}{4b}\right) \left(\frac{(1-t-c)(-2+2t+1-t-c)}{(1-t)^2}\right) \\
&= \left(\frac{1}{4b}\right) \left(\frac{(1-t-c)(t-1-c)}{(1-t)^2}\right) \\
&= -\left(\frac{1}{4b}\right) \left(\frac{(1-t-c)(1-t+c)}{(1-t)^2}\right)
\end{aligned}$$

But since $(1-t-c)(1-t+c) = (1-t)^2 - c^2$, we end up with

$$\begin{aligned}
\pi'(t) &= -\left(\frac{1}{4b}\right) \left(\frac{(1-t)^2 - c^2}{(1-t)^2}\right) \\
&= -\frac{R(t)}{t}
\end{aligned}$$

where we have used (5).

7) To find the derivative of the Nash product, we derive (14) using the quotient rule. We have

$$N(t) = \left(\frac{1}{4b}\right)^2 \left(\frac{((1-t)-c)}{1-t}\right)^3 t((1-t)+c)$$

and so

$$\begin{aligned}
N'(t) &= \left(\frac{1}{4b}\right)^2 \left[3 \left(\frac{1-t-c}{1-t}\right)^2 \left(\frac{-c}{(1-t)^2}\right) t(1-t+c) \right. \\
&\quad \left. + \left(\frac{1-t-c}{1-t}\right)^3 (1-2t+c) \right]
\end{aligned}$$

Multiply the second term in square brackets by $\frac{1-t}{1-t}$, and then take out the common factors;

$$N'(t) = g(t) [-3ct(1-t+c) + (1-t)(1-t-c)(1-2t+c)]$$

where $g(t) \equiv \left(\frac{1}{4b}\right)^2 \left(\frac{1-t-c}{1-t}\right)^2 \left(\frac{1}{(1-t)^2}\right)$. This is the equation given in the text.

8) The ratio of profit to copyright income, for any t , is

$$\frac{\pi(t)}{R(t)} = \left(\frac{1-t}{t}\right) \left(\frac{1-t-c}{1-t+c}\right)$$

Let's work this out term by term for the case of

$$t = t^{Ne} = \left(\frac{1}{2}\right) \left(\frac{1 - c^2}{2 + c + c^2}\right)$$

Firstly, we have $1 - t^{Ne}$ equal to

$$\begin{aligned} 1 - \left(\frac{1}{2}\right) \left(\frac{1 - c^2}{2 + c + c^2}\right) &= \frac{2(2 + c + c^2) - (1 - c^2)}{2(2 + c + c^2)} \\ &= \frac{4 + 4c + 4c^2 - 1 + c^2}{2(2 + c + c^2)} \\ &= \frac{3 + 4c + 5c^2}{2(2 + c + c^2)} \end{aligned}$$

Thus, the ratio of $1 - t^{Ne}$ to t^{Ne} is

$$\frac{1 - t}{t} = \frac{3 + 4c + 5c^2}{1 - c^2}$$

Second, we need to work out both $1 - t^{Ne} - c$ and $1 - t^{Ne} + c$. The first of these is

$$\begin{aligned} 1 - t^{Ne} - c &= \frac{3 + 4c + 5c^2}{2(2 + c + c^2)} - c \\ &= \frac{3 + 4c + 5c^2 - c2(2 + c + c^2)}{2(2 + c + c^2)} \\ &= \frac{3 + 4c + 5c^2 - 4c - 2c^2 - 2c^3}{2(2 + c + c^2)} \\ &= \frac{3 + 3c^2 - 2c^3}{2(2 + c + c^2)} \end{aligned}$$

And the second of them is

$$\begin{aligned} 1 - t^{Ne} + c &= \frac{3 + 4c + 5c^2}{2(2 + c + c^2)} + c \\ &= \frac{3 + 4c + 5c^2 + c2(2 + c + c^2)}{2(2 + c + c^2)} \\ &= \frac{3 + 4c + 5c^2 + 4c + 2c^2 + 2c^3}{2(2 + c + c^2)} \\ &= \frac{3 + 8c + 7c^2 + 2c^3}{2(2 + c + c^2)} \end{aligned}$$

Therefore, we get

$$\frac{1-t-c}{1-t+c} = \frac{3+3c^2-2c^3}{3+8c+7c^2+2c^3}$$

and so,

$$\frac{\pi}{R} = \left(\frac{3+4c+5c^2}{1-c^2} \right) \left(\frac{3+3c^2-2c^3}{3+8c+7c^2+2c^3} \right)$$

which is the equation given in the text.