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COPYRIGHT ENFORCEMENT AND QUALITY DIFFERENTIATION ON THE INTERNET

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Abstract. Right-holders can create differences between their cultural goods to attract consumers with varying levels of willingness to pay. Some Internet intermediaries propose similar choices but do so without authorization. In this paper, we present a theoretical model of copyright piracy in which a right-holder competes in price with an Internet intermediary in a leader-follower game. The Internet intermediary provides two types of streaming goods (with and without restrictions). Copyright and intellectual property rights on the Internet are subject to ex-post adjudication. This means that enforcement can lead to uncertainty regarding Internet intermediaries' liability. We analyze how copyright enforcement and quality differences impact price competition. Our analysis suggests that law uncertainty plays a role in an intermediary's decision to enter the market, and thus that quality has an impact on law enforcement efficiency.

1. Introduction

The legal supply of cultural works, both online (e.g. video on demand (VOD), streaming music via Spotify or Deezer, etc.) and on other media (e.g. CD, DVD, Blu-Ray), is subject to competition from Internet intermediaries[1] supplying illegal streaming or downloading, i.e. without the consent of the right-holder.

On the digital marketplace, this competition is reflected in the segmentation of supply. Whether supplied legally or illegally, the various files may have specific characteristics (i.e. audio, image, download speed, and streaming technology[2]) that are both technical and related to that particular work's contextual environment. Individual works are available in different formats with specific features, e.g. films or music are sold on physical media, but can also be legally purchased in dematerialized format. In addition, it is possible to buy a single work and add editorial

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[1] "Internet intermediaries bring together or facilitate transactions between third parties on the Internet. They give access to, host, transmit and index content, products and services originated by third parties on the Internet or provide Internet-based services to third parties" (OECD, 2011).

[2] Streaming technology is a means of delivering a media with partial storage: end-users can start playing or visualizing a work before the entire file has been downloaded on their computer.

supplements. The idea is to adapt the work to a particular environment – depending on the medium through which it will be viewed or listened to, to match the tastes of the audience, etc. – and encourage positive externalities between various artistic creations. Moreover, the differentiation is temporal since audiovisual works introduced onto the market follow a specific timeline. Internet intermediaries can benefit from this time constraint, supplying audiovisual works in advance.

This segmentation of supply goes hand in hand with various business models: advertising for free access and/or different types of paid access. The latter could include subscriptions for unlimited access to the files or time-limited purchases. A file hosted by an intermediary without authorization may be viewed for free via streaming but with a limited viewing time, whereas with a subscription the user may have unlimited access to all stored files with a faster download speed. We call this process commercial piracy. Legal intermediaries can also supply subscriptions providing access to a certain number of works e.g. VOD by subscription, or purchases may cover a given period of time (e.g. film in digital format visible for one month from its rental date).

Segmentation of supply can be viewed as quality differentiation (also called an "editorialization" process), which is a key parameter in this competition framework. Quality reflects physical specificities, supplements, and the availability of different cultural works (especially movies).

In legal terms, these intermediaries are not responsible a priori for the legality of files they host on their platform: their liability depends on the legal recognition of their status as host and judicial decisions in this field. They are not held liable as long as there is no evidence of their knowledge of the unlawful status of a file.[3] This raises uncertainty regarding the application of their status. Hence, liability is decided by trial and the judge's interpretation.

The issue at hand is the following: how, in an environment that values the liability of host websites (albeit at the judge's discretion), is quality competition reflected between the legal and illegal distribution of audiovisual works? More specifically, what is the relationship between the choice of quality of the works on supply and the intermediaries' civil liability?

The originality of our approach is to apply tort law to copyright enforcement on the Internet. This application is linked to the hosting status of Internet intermediaries: their responsibility is based[4] on the difference between the functions of host and editor. The former stores files belonging to others, whereas the latter modifies and shapes the media available on its platform. The fact that audiovisual works

[3]This is defined in the Digital Millennium Copyright Act (1998) in the United States, and the Electronic Commerce Directive (2000) in Europe.

[4]In France, this status is defined in the "Loi pour la Confiance dans l'économie numérique", (Loi n°2004-575), which introduced this distinction.

can be viewed in streaming mode without the authorization of right-holders puts the accent on intermediaries' responsibility. Hence, they can now be sued for their responsibility as a host. Numerous law cases regarding intellectual property rights (e.g. brand, copyright) question this status.

Therefore, the aim of this paper is twofold. First we model competition between legal and illegal supply. Then we examine two types of relationship. On the one hand, we study the impact of quality on equilibrium prices. On the other hand, we analyze the efficiency of law enforcement in restricting commercial piracy on the Internet when there is quality differentiation.

To address this issue, we present a theoretical model of copyright piracy in which a right-holder competes in price with an Internet intermediary. The copyright holder can sue the intermediary which supplies files without authorization. The legal authority has to decide on the liability of the intermediary.

Internet intermediaries supply (and host) pirated content e.g. through streaming technology, in one of the following two ways (versioning of the good): consumers can benefit from free content but with restrictions (e.g. viewing time, as on Megavideo[5]), or they can buy unlimited access reflecting the segmentation in supply. Among the intermediaries that facilitate streaming, several categories can be distinguished:[6]

- Host sites that enable direct download or use of streaming without specific research tools. Some sites provide only one of these two functions;
- Referral sites that provide links to download files or watch films in streaming mode hosted on other platforms.

The Internet intermediaries studied in our paper belong to the first category.

As pointed out by Mussa and Rosen (1978), there is a quality differentiation, since the streaming goods supplied by the intermediary could be of lower quality than the legal ones. For example, there is a quality difference between a movie watched from a DVD and one watched directly on the Internet (audio, image, editorial supplements etc.). Hence, the quality parameters indicate the quality of the files. Furthermore, a double quality differentiation exists here, since the intermediary is supplying two distinctive goods.

Consequently, the quality of a legal good can change, which means that copyright-holders are able to differentiate their own goods. The quality of unlimited illegal products is superior to that of goods with restrictions (e.g. no limitation on the

[5]Part of the Megaupload "galaxy", which was closed down by the US Justice Department in January 2012.

[6]*Etude du modèle économique de sites ou services de streaming et de téléchargement direct de contenus illicites*, Report for the attention of the Hadopi, IDATE 2012.

contents visualized) but they may have the same value as legal products. The quality initially conferred on products by their legal distributors can be improved by differentiation implemented by both legal agents and illegal intermediaries: in the case of music, platforms can supply songs to listen to on playlists and a unique visual environment; access to a high number of films or series on illegal platforms can raise the quality of membership compared to that of authorized distribution. The quality of streaming products can also stem from the fact that some audiovisual works are available in streaming mode before they become available through legal means. This is the case for some foreign television series. We choose to study this competition in a leader-follower price game in which quality is exogenous.

We show that pricing and the level of law enforcement required to evict Internet intermediaries are both determined in relation to quality. We also use a comparative statics analysis to demonstrate the link between quality choices and the intermediary's liability: the latter's investment in quality is linked to the maximum probability (or legal strength) that allows the intermediary to remain in business. Therefore, the law efficiency parameter is not unique and is linked to quality parameters. But we find that for some market cases (when the intermediary only supplies one good), this probability is independent of quality.

The paper is structured as follows. Section 2 reviews related literature and specifies what makes our framework distinctive. In section 3 we present the model and the main variables. Section 4 examines the equilibrium and the influence of legal enforcement on intermediaries' decisions. In section 5 we conduct a comparative statics analysis. Conclusions are provided in section 6.

2. Related literature

The main approach here is to apply tort law to Internet intermediaries that supply streaming goods: they can be held liable for infringing intellectual property law. This is a new perspective in literature. As mentioned above, we focus on the hosting status of such websites. Beginning with Shavell (1984a, 1984b and 1987) and Landes and Posner (1987), the literature on tort law studies how economic agents internalize the costs of externalities that could be caused by their actions. Economics and the law come together in the search for efficient behavior that minimizes the social cost of a potential tort by internalizing this externality.

Numerous rules of liability have been studied, taking into account the liability between victim and injurer (strict liability, negligence rule). This liability can in some cases be shared between parties (contributory and comparative negligence). In the formalization of such a system, agents take action (i.e. "care") to avoid accidents. Thus, convictions are not systematic and courts make decisions based on their interpretation of social optimum levels of precaution (for negligence rules).

Tort analysis has been extended to errors in legal operations (Dari-Mattiacci, 2005), e.g. in a due care situation. Uncertainty in the legal system makes the issue of conflict resolution difficult to anticipate, as in our present case regarding the liability of intermediaries.

Focusing on copyright law, Arai (2011) compares the involvement of civil and criminal penalty schemes (i.e. penalties paid to the copyright holder or the government) from the viewpoint of social welfare in cases of copyright violation. Martínez-Sánchez (2010) analyzes the role of the government and a legal producer in preventing the entry of a pirate in a sequential duopoly model of vertical product differentiation. The latter can give the pirate the advantage of setting the price first. Banerjee (2006) studies the effect of enforcement sharing between the government and the incumbent in a commercial piracy framework (the former penalizes and the latter monitors). Government sensitivity to piracy is an important condition for preventing infringement.

Arai's analysis (2011) is more closely related to our framework since he studies the interaction between copyright holders, copyright infringement and the legal penalty.

We link tort law to commercial piracy. This is a new approach since not long ago Internet intermediaries were protected by their hosting status. Recently, however, legal decisions have questioned this position.

Our model also tries to capture quality competition between an Internet intermediary and a right-holder. Two papers are related to this issue: Banerjee (2003) studies competition between a copyright owner and a pirate who tries to enter the market, and the government's role in penalizing piracy. He finds that if monitoring is the optimal policy, then a monopoly situation results. Kiema (2008) extends this analysis to the competition between a monopolist and several commercial pirates.

3. The model

We build a model that describes price competition between a right-holder and an Internet intermediary that supplies illegal products. Products are differentiated in quality. We consider that there is an exogenous probability that the intermediary is found liable and has to pay a fine to the copyright holder. This framework enables us to study how competition is altered by legal uncertainty and quality differentiation. In this section, we first present the behaviors and parameters of the main actors of our framework, and then the demand for legal and illegal supply.

3.1. **Legal setting.** Regarding the legal copyright framework for Internet intermediaries, the court has to decide on the ex-post liability of these websites. We are in a civil law set-up, meaning that there is only a monetary transfer from the copyright infringer to the copyright holder.

Following the discovery of a pirate or a streaming website supplying illegal files, the legal beneficiary can bring the case to court and the right-holder can demand financial compensation. Subsequently, the judge has to determine the liability of the Internet intermediary (i.e. was it aware of the infringement?). Due to the uncertainty of law enforcement in this area (e.g. different decisions depending on the case, judge, etc.), the intermediary is declared responsible and has to pay the copyright holder compensation with a probability of q.[7] q represents the strength of the copyright law from the law-maker's point of view as well as uncertainty regarding law enforcement (as explained in the introduction) from the intermediary's point of view. The penalty paid is proportional to the demand that the intermediary receives. This is defined by G multiplied by the demand, with $G \in [0, 1]$. q can be seen as the severity of the law whereas G is linked to the intermediary monetary value. We refer to qG as the "expected marginal fine".

3.2. **Legal supply.** We suppose that there is only one monopolist producer (or right-holder) of the legal good with quality $a \in [0, 1]$ which it sells at price p. This can be justified by arguing that cultural goods are sufficiently horizontally differentiated to make the demand independent from the price of other goods in the same category. When an Internet intermediary is found guilty, the copyright holder receives the expected penalty paid. For the sake of simplicity, we also take production costs to be nil.

Moreover, we make an extreme assumption in our framework: we suppose that the Internet intermediary is always sued and that there is no private settlement. However, for the most part, going to court is costly for the right-holder.

3.3. **Internet intermediary.** The Internet intermediary supplies two types of illegal streaming, i.e. streaming of pirated goods. There is only one Internet intermediary in our framework. The Internet intermediary has two sources of revenue: the price paid for unlimited content access with quality b, and advertising[8] revenue normalized to 1 generated by demand for the free version with quality c.

Assumption 1: Here $0 < c < b < a \leq 1$.

Assumption 1 defines quality for the legal or illegal goods. These are all exogenous parameters in our analysis.

[7] $q \in [0, 1]$.

[8] On this type of website, advertising comes mainly in the form of pop-ups and banners (*Etude du modèle économique de sites ou services de streaming et de téléchargement direct de contenus illicites*, Report for the attention of the Hadopi, IDATE 2012).

The Internet intermediary is found liable with probability q. In such case, it has to pay a fine proportional to its demand. We also take reproduction costs to be nil.

3.4. **Consumer demand.** There is a continuum of consumers indexed by θ who value the digital good differently. θ also represents their willingness to pay and it is uniformly distributed on $[0, 1]$.[9]

Consumers have three options: First they can purchase the good legally at price p. Second, they can use it freely on the Internet but with restrictions. Lastly, they can buy unlimited access to the streaming website and its contents at a price p_i, which can be understood as a subscription fee. Users do not face risk of prosecution from using streaming websites.

Consumer utility is defined as follows:

$$U = \begin{cases} a\theta - p & \text{if the consumer buys the legal product} \\ b\theta - p_i & \text{if the consumer pays to have unlimited access} \\ c\theta & \text{if the consumer uses streaming with restriction} \end{cases}$$

What is the demand for the different goods? We need to distinguish between two different cases depending on the parameters.

In Figure 1 we show the utility function for each case. In the figure, we have drawn two different utility lines for the case of a consumer purchasing the legal product, in order to easily point out the feature of the parameters that fully defines the demand scenarios. Specifically, we have drawn a legal good utility with price p_1, and another with price $p_2 > p_1$. So begin by looking at the utility for the restricted online product, $U = c\theta$ along with the utility for the online subscription product, $U = b\theta - p_i$. Since $b > c$, these two lines must intersect at some point, which is labelled as point B. The corresponding level of θ is denoted θ_{ru}, which is the level of θ such that the restricted online product and the online subscription product are equally preferred. We assume that $\theta_{ru} < 1$, or else only restricted free access is demanded from the intermediary for all valid values of θ.

Now, we superimpose upon the figure the utility of consuming the legal product, $U = a\theta - p$. Since $a \geq b$, we draw this line strictly steeper than $U = b\theta - p_i$. Start by assuming that $p = p_1$, such that $U = a\theta - p$ passes above the point B. This utility function intersects the utility $U = c\theta$ at the point A. The corresponding level of θ is denoted as θ_{lr}, and it is the level of θ such that the consumer is indifferent between the free online product and the legal product. Notice that with this price for the legal product, the consumer would consume the free online product for all $\theta < \theta_{lr}$, and the legal product for all $\theta \geq \theta_{lr}$. In this case, the online subscription product is not demanded at all. We refer to this sort of situation as "case 1".

[9]We assume that the market is always covered by either the legal good or the streaming good.

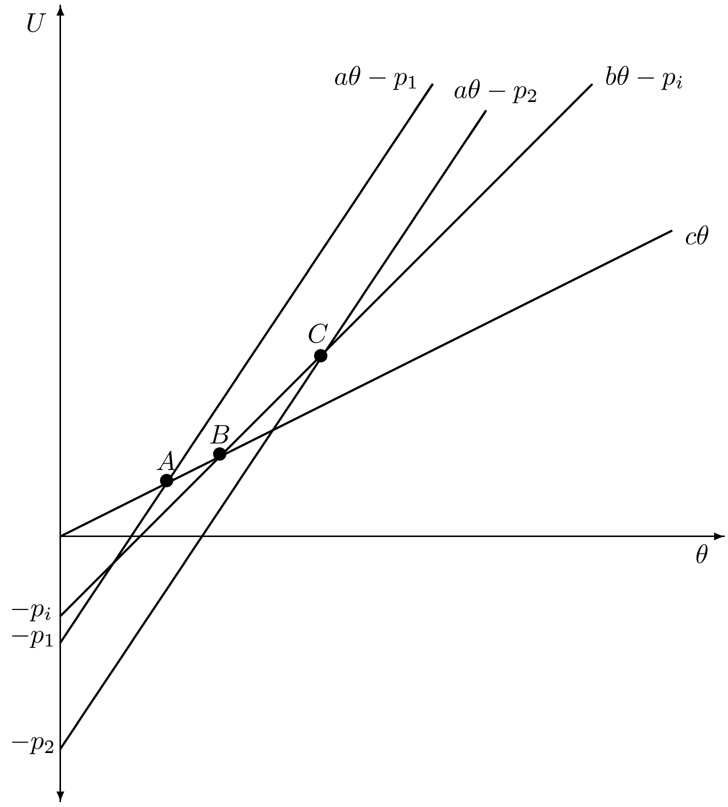

Figure 1: The utility function

Second, assume that $p = p_2$, where $p_2 > p_1$, such that $U = a\theta - p$ passes below the point B. This utility function intersects the utility $U = b\theta - p_i$ at the point C. The corresponding level of θ is denoted as θ_{lu}, and it is the level of θ such that the consumer is indifferent between the online subscription product and the legal product. With this price for the legal product, the consumer would consume the free online product for all $\theta < \theta_{ru}$, the online subscription product for θ such that $\theta_{ru} \leq \theta < \theta_{lu}$, and the legal product for all $\theta \geq \theta_{lu}$, so long as $\theta_{lu} \leq 1$.[10] In this case, there is demand for all three products. We refer to this situation as "case 2".

We can easily calculate the three critical values of θ:

$$a\theta_{lr} - p = c\theta_{lr} \Rightarrow \theta_{lr} = \frac{p}{a-c}$$

$$c\theta_{ru} = b\theta_{ru} - p_i \Rightarrow \theta_{ru} = \frac{p_i}{b-c}$$

[10]The legal supplier can set p so high as to eliminate all demand for his product, by pricing such that $\theta_{lr} \geq 1$. However it is clear that this will never actually happen, since it would generate profit of 0 when positive profits are certainly possible. So p will always be set such that $\theta_{lr} < 1$, which we also note is always possible since we started off by assuming that $\theta_{ru} < 1$.

$$a\theta_{lu} - p = b\theta_{lu} - p_i \Rightarrow \theta_{lu} = \frac{p - p_i}{a - b}$$

The defining characteristic between cases 1 and 2 is the comparison between θ_{lr} and θ_{ru}, i.e. the comparison between $\frac{p}{a-c}$ and $\frac{p_i}{b-c}$. Thus, it happens that case 1 is when $\frac{p}{a-c} \leq \frac{p_i}{b-c}$, which is $p \leq \frac{p_i(a-c)}{b-c}$, and case 2 is $p > \frac{p_i(a-c)}{b-c}$.

All of this can be summed up in the following Lemma:

Lemma 1. *There are two cases;* ***Case 1****;* $p \leq \frac{p_i(a-c)}{b-c}$*. In this case the restricted illegal product is demanded for all* $\theta < \theta_{lr} = \frac{p}{a-c}$*, and for all* $\theta \geq \frac{p}{a-c}$ *the legal product is demanded. In case 1 there is no demand for the subscription product.* ***Case 2****;* $p > \frac{p_i(a-c)}{b-c}$*. In this case it must hold that* $\theta_{ru} < \theta_{lr}$*. In this case the restricted illegal product is demanded for all* $\theta < \theta_{ru} = \frac{p_i}{b-c}$*, the unrestricted (subscription) illegal product is demanded for all* θ *such that* $\theta_{ru} \leq \theta < \theta_{lu}$*, and the legal product is demanded for all* $\theta \geq \theta_{lu}$*, so long as* $\theta_{lu} < 1$*.*

The strategic variables here are prices. We consider that the quality parameters (a, b and c), the strength of the law q, and G are all exogenous.

The sequence of events is:

(1) The government chooses G and an enforcement level (or legal uncertainty) q.
(2) The producer and the intermediary compete in a leader-follower game and choose their prices p_i and p.
(3) The consumer decides whether to buy the legal good, use it illegally with restrictions, or pay for unlimited access.
(4) Any intermediary who is caught and found guilty has to pay the penalty G proportional to the demand.

The law is constructed ex-ante and the law-maker chooses the severity of the rule. The judge enforces the law ex-post according to his or her interpretation (as in tort law).

4. Equilibrium

In this section, we examine pricing games between the Internet intermediary and the copyright holder for cases 1 and 2 described above. We consider quality choices (a, b, c) as exogenous. We derive the equilibrium prices in each situation.

We are in a leader-follower game (Stackelberg): the incumbent (i.e. the legal developer) takes into consideration that an intermediary will enter and therefore incorporates the reaction function of this agent into its profit function and chooses the profit-maximizing price. We have chosen this type of competition since it is the most common in real life. We work backwards to solve the problem based on the demand described in the previous section.

Denoting by D_u the demand for unlimited access and by D_r the demand for free access, the Internet intermediary's profit is defined as follows:

$$\pi_i = p_i D_u + D_r - qG(D_u + D_r) \tag{1}$$

$qG(D_u + D_r)$ is the expected fine the intermediary has to pay if found guilty. The fine is based on the total demand $D_u + D_r$. If $G = 1$, the fine is the value of the intermediary's total demand, or in the other words, the market that the right-holder cannot access because of competition with the intermediary.

The right-holder's profit function is:

$$\pi_l = pD_l + qG(D_u + D_r) \tag{2}$$

where D_l is the demand for the legal product and $qG(D_u + D_r)$ the expected compensation paid by the copyright infringer if it is discovered and found guilty by the judge.

We first solve out for the equilibrium pricing in case 2, and then we solve the equilibrium for case 1.

4.1. **Case 2.** In case 2, there is some demand for all three products. In this case, equation (1) can be written as:

$$\begin{aligned}\pi_{i,2} &= p_i(\theta_{lu} - \theta_{ru}) + \theta_{ru} - qG\theta_{lu} \\ &= p_i\left(\frac{p - p_i}{a - b} - \frac{p_i}{b - c}\right) + \frac{p_i}{b - c} - qG\left(\frac{p - p_i}{a - b}\right)\end{aligned}$$

The right-holder's profit function for case 2 is:

$$\begin{aligned}\pi_{l,2} &= p(1 - \theta_{lu}) + qG\theta_{lu} \\ &= p\left(1 - \frac{p - p_i}{a - b}\right) + qG\left(\frac{p - p_i}{a - b}\right)\end{aligned}$$

When the intermediary and the right-holder are in price competition, they take each other's reaction function into account. In a leader-follower game, the latter incorporates the intermediary's reaction-function into its profit function. By solving the first order conditions, we find the equilibrium prices as given by Proposition 1.

Proposition 1. *The equilibrium prices are:*[11]

$$\begin{aligned}p_2^* &= \frac{(a - b)(2(a - c) + 1) + 2qG(a - c)}{2(2a - c - b)} \\ p_{i,2}^* &= \frac{a - b + qG(b - c)}{2(a - c)} + \frac{b - c}{2(a - c)}\frac{(a - b)(2(a - c) + 1) + 2qG(a - c)}{2(2a - c - b)}\end{aligned}$$

Proof. See Appendix A-1. □

[11]By assumption 1, we have $2a - c - b = (a - c) + (a - b) > 0$, so the prices are positive.

We note that equilibrium prices take into account all quality parameters as well as the expected marginal fine, qG.

4.1.1. *Equilibrium existence.* This equilibrium only works if the constraint $p > \frac{p_i(a-c)}{b-c}$ is satisfied. Several results are in order regarding existence of the case 2 equilibrium. To begin with, we have:

Proposition 2. *A necessary and sufficient condition for a case 2 equilibrium to exist is*

$$b < \frac{2(a-c)(1+c)+2a+c}{2(a-c)+3}$$

Proof. See Appendix A-2. □

So if the quality parameters satisfy Proposition 2, we know that there will be some values (specifically, smaller values) of qG for which a case 2 equilibrium must exist. But we can also identify the limit value of the expected marginal fine for which the equilibrium will exist. Specifically;

Proposition 3. *In any case 2 equlibrium, the expected marginal fine must satsify*

$$qG < \frac{1}{2} + (a-c) - \frac{(a-c)+(a-b)}{b-c}$$

Proof. See Appendix A-3. □

As a corollary, in any case 2 equilibrium it must necessarily hold that $qG < \frac{1}{2}$. To see this, continue from Propostion 3 and note that:

$$(a-c) - \frac{(a-c)+(a-b)}{b-c} = (a-c)\left(1 - \frac{1}{b-c}\right) - \frac{a-b}{b-c} < 0$$

where the inequality follows from Assumption 1, which implies that $b-c < 1$. Thus

$$\frac{1}{2} + (a-c) - \frac{(a-c)+(a-b)}{b-c} < \frac{1}{2}$$

As an example, set $a = 1$ and $c = 0.1$. It turns out (from Proposition 2) that a case 2 equilibrium exists for values of b greater than 0.85. Specifically, (from Proposition 3) for $b = 0.9$, there is a case 2 equilibrium for all values of qG less than 0.15.

4.2. **Case 1.** What happens if the constraint in Proposition 2 does not hold, that is, the legal supplier's price and the intermediary's price for the subscription product do not satisfy the constraint $p > \frac{p_i(a-c)}{b-c}$? Then the case 2 equilibrium does not hold, and instead we are in case 1, in which there is no demand for the subscription product. The consumer can only buy the legal good, or use free streaming access. This is because unlimited access is too expensive.

In this case the copyright holder's profit function is (from equation (2)):

$$\begin{aligned}\pi_{l,1} &= p(1-\theta_{lr}) + qG\theta_{lr} \\ &= p\left(1-\frac{p}{a-c}\right) + qG\frac{p}{a-c}\end{aligned}$$

The Internet intermediary's profit function is (from equation (1)):

$$\begin{aligned}\pi_{i,1} &= \theta_{lr} - qG\theta_{lr} \\ &= (1-qG)\frac{p}{a-c}\end{aligned}$$

The intermediary only earns revenue from advertising because there is no demand for unlimited access in this case. In solving the first order condition, we find:

Proposition 4. *The right-holder's equilibrium price is:*

$$p_1^* = \frac{a-c+qG}{2}$$

Proof. A proof is provided in Appendix A-4. □

In this case, the market is not shared equally. We find that $\theta_{lr} = \frac{1}{2} + \frac{qG}{2(a-c)}$. Demand for the intermediary is θ_{lr}, which means that it possesses more than half of the market. Even though the quality of free access is lower than that of the legal good, the intermediary has a higher market share. Here "quality" is dominated by "price": more consumers choose the free, lower quality good, rather than the legal, costly good.

Moreover, the expected marginal fine, qG, plays a role in the market share. If qG increases, somewhat surprisingly the intermediary's market share rises. When qG increases, the equilibrium price of the legal supplier is higher and the demand is lower. At the same time, the demand for the intermediary's products increases.

Substituting the optimal legal supply price into the intermediary's profit, we find that

$$\pi_{i,1} = (1-qG)\frac{(a-c+qG)}{2(a-c)}$$

and the legal supplier's profit is

$$\pi_{l,1} = \left(\frac{a-c+qG}{2}\right)\left(1-\frac{a-c+qG}{2(a-c)}\right) + qG\left(\frac{a-c+qG}{2(a-c)}\right)$$

which reduces to:

$$\pi_{l,1} = \frac{(a-c+qG)^2}{4(a-c)}$$

The legal supplier's profit is an increasing convex function of qG, so the higher is the expected marginal fine the better for the legal supplier. On the other hand, the intermediary's profit is non-monotone in qG. Specifically,

$$\frac{\partial \pi_{i,1}}{\partial (qG)} = -\frac{a-c+qG}{2(a-c)} + \frac{1-qG}{2(a-c)}$$

$$\frac{\partial^2 \pi_{i,1}}{\partial (qG)^2} = -\frac{1}{2(a-c)} - \frac{1}{2(a-c)} = -\frac{1}{a-c} < 0$$

Thus the intermediary's profit is a concave function of the expected marginal fine. $\pi_{i,1}$ goes to 0 when $qG = 1$, and $\pi_{i,1} > 0$ for all qG such that $0 \leq qG < 1$. Finally, $\pi_{i,1}$ is increasing in qG up to $qG = \frac{1}{2} - \frac{a-c}{2} > 0$, and decreasing in qG for larger values of the expected marginal fine.

5. Comparative statics

Having described the demand, and the equilibrium prices, we go on to focus on comparative statics. How do variations in quality affect the equilibrium?

5.1. **Equilibrium prices.** We study how equilibrium prices change if the quality of the three goods supplied in the market changes. It is easy to obtain the following comparative statics:

Proposition 5. *In case 2, the right-holder equilibrium price increases with a and decreases with b. It increases (resp. decreases, does not change) with c as $qG <$(resp. $>, =$)$\frac{1}{2} - (a-b)$.*

Proof. See Appendix B-1. ☐

When the quality of the legal good a increases, the right-holder's equilibrium price goes up to take this new value into account. Improving quality is a way for the right-holder to differentiate its good even further from that of the intermediary. However, when the quality supplied by the intermediary increases, the result is less straightforward. First, when b increases, the right-holder has an incentive to decrease its price to remain competitive. This is because b approaches a. Secondly, when the quality of the limited good c goes up, the same relationship appears: c approaches b and a, and can be seen by the right-holder as a threat. Therefore, the legal supplier chooses to decrease its price if the expected fine is sufficiently high.

Regarding the intermediary equilibrium's behavior, we expect the following result: $p^*_{i,2}$ decreases with a and increases with b or c. This comes from the competition game between the two actors. We establish a new proposition for the intermediary:

Proposition 6. *In case 2, the relationship between the intermediary's equilibrium price and quality parameters is generally non-monotone, and is conditional on the value of p^*_2. Specifically, it happens that (a) $\frac{\partial p^*_{i,2}}{\partial a} > 0$ if $qG \leq (a-c)\frac{\partial p^*_2}{\partial a} - p^*_2$, (b) $\frac{\partial p^*_{i,2}}{\partial b} \gtreqless 0$ as $qG \gtreqless 1 - (b-c)\frac{\partial p^*_2}{\partial b} - p^*_2$, and (c) $\frac{\partial p^*_{i,2}}{\partial c} > 0$ if $qG < \frac{1}{2} - (a-b)$.*

Proof. See Appendix B-2. □

The effects upon the intermediary's equilibrium price as a and b change depend upon the values of differential equations of the right-holder's equilibrium price. In essence, any signs can be generated with different parameter sets. Combining the previous two propositions, we can say that if $qG < \frac{1}{2} - (a - b)$ then both the right-holder's price and the intermediary's price will increase with c.

We now turn to case 1, in which the Internet intermediary only supplies the free streaming good on the market. The right-holder sets the price

$$p_1^* = \frac{a - c + qG}{2}$$

Clearly, this is increasing in a and decreasing in c. So if the quality of the free good rises, the legal supplier reduces the price in response in order to retain a competitive edge for high θ demanders.

5.2. **Effects on profits.** In case 1 we can also easily find the effects upon the profits of each player of changes in a and c.

Proposition 7. $\frac{\partial \pi_{l,1}}{\partial a} \gtreqless 0$ *as* $qG \gtreqless a - c$, *and* $\frac{\partial \pi_{l,1}}{\partial c} \gtreqless 0$ *as* $qG \lesseqgtr a - c$.

Proof. See Appendix B-3. □

Thus, the legal supplier's profit rises with one of a and c, and decreases with the other, and which of the two is beneficial depends on the relationship between qG and $a - c$. Above all, we note that it is possible that the legal supplier's profit is reduced by an increase in the quality of the legal good, and increased by an increase in the quality of the intermediary's good.

On the other hand, the results for the intermediary's profit are much more expected:

Proposition 8. $\frac{\partial \pi_{i,1}}{\partial a} \leq 0 \leq \frac{\partial \pi_{i,1}}{\partial c}$.

Proof. See Appendix B-4. □

Therefore an increase in a decreases the intermediary's profit, while an increase in c increases the intermediary's profit.

The effects upon the profits of the two players in case 2 will not be attempted here, as they will of course be even more complex than the price effects.

5.3. **Demand.** We study now the impact of quality choices on piracy. We use the equilibrium Internet intermediary demands as a measure of piracy. In either case 1 there is no demand for the subscription product, and the right-holder's market share is $1 - \theta_{lr}^*$, while the intermediary's market share is θ_{lr}^*. On the other hand, in case 2 the intermediary's market share is θ_{lu}^* (which is split between the free product, up to θ^*_{ru} and the subscription product between θ^*_{ru} and θ^*_{lu}) and the right-holder's market share is $1-\theta^*_{lu}$. We already know the following;

$$\theta_{lr} = \frac{p^*_1}{a-c}, \quad \theta_{ru} = \frac{p^*_{i,2}}{b-c}, \quad \theta_{lu} = \frac{p^*_2 - p^*_{i,2}}{a-b}$$

It is easiest to start with case 1, where only θ_{lr} is of issue. We get:

Proposition 9. *An increase in a reduces piracy in case 1. An increase in c increases piracy in case 1.*

Proof. See Appendix B-5. □

Case 2 is much more complex, as the piracy effects depend upon the price effects of changes in the quality parameters, which above we saw were not clear-cut. We need to investigate the effects of changes in the three quality variables upon $\theta_{lu} = \frac{p^*_2 - p^*_{i,2}}{a-b}$. Clearly, all three effects will depend upon how the difference between the two prices is affected. Concretely, the results are the following:

Proposition 10. *(a)* $\frac{\partial \theta_{lu}}{\partial a} < 0$. *(b)* $\frac{\partial \theta_{lu}}{\partial b} > 0$. *(c)* $\frac{\partial \theta_{lu}}{\partial c} > 0$.

Proof. See Appendix B-6. □

In short, an increase in the quality of the legal good, a, will decrease piracy, and increases in the quality of either of the intermediary's goods, b or c, will increase piracy.

6. Conclusions

This paper explores the strategic behavior of a copyright-holder and an Internet intermediary. The latter supplies two types of goods, one of which is restricted and one of which is not. We model a situation involving only ex-post adjudication. This situation leads to uncertain legal enforcement. It corresponds to the concept of a "safe harbor" for Internet intermediaries.

First, we show that right-holder prices take into account the quality of the Internet intermediary's products. Moreover, raising the quality parameters of illegal streaming products can in some cases increase the Internet intermediary's market share.

Second, two types of relation can be explored. First, equilibrium prices and quality are related to each other and together they define the competition. Then, the effectiveness of legal enforcement depends on quality parameters and market cases. Surprisingly, when the intermediary only supplies the free access good, the intermediary's profit is non-monotone in the expected marginal fine: its profit is increasing in qG up to a unique point, which is defined according to quality.

These results have policy implications for copyright rules and innovation in the field of legal content supplied on the Internet. In our analysis the expected marginal fine and thus the enforcement level are a function of the version strategies chosen by the Internet intermediary and the right-holder. We have shown that the required enforcement level that reduces intermediary profit to zero is a function of the version strategies chosen by the Internet intermediary and the right-holder. Conversely, right-holders and Internet intermediaries must take current legislation into consideration. Furthermore, the extent and cost of private monitoring raise genuine issues regarding the efficiency of legal procedures.

We consider possible avenues for extending our results. For a more general representation, we could extend this analysis to cover private enforcement (i.e. monitoring). In particular, copyright holders could be obliged to enforce their right (e.g. notice and take down procedure) and therefore make an effort. The intermediary can also act in reducing the probability of being caught and fined. Moreover, some assumptions could be revised to extend our model, such as endogenous quality, other tort law rules (like strict liability), separation of legal and private enforcement (by right-holders).

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Appendix

Appendix A: Equilibrium

Appendix A-1: Proof of Proposition 1.

The intermediary's reaction function is:

$$p_i(p) = \frac{p(b-c)}{2(a-c)} + \frac{a-b+qG(b-c)}{2(a-c)}$$

We substitute the Internet intermediary's reaction function into the right-holder's profit function, equation (2), to obtain the optimal prices. These are maximum prices since:

$$\begin{aligned}\frac{\partial^2\pi_{l,2}}{\partial p^2} &= -\frac{2}{a-b} < 0\\ \frac{\partial^2\pi_{i,2}}{\partial p_i^2} &= -\frac{2}{a-b} - \frac{2}{b-c} < 0\end{aligned}$$

Appendix A-2: Proof of Proposition 2.

Notice (from Proposition 1) that both p_2^* and $p_{i,2}^*$ are linear functions of qG. Therefore $\frac{p_i(a-c)}{b-c}$ is also a linear function of qG. Define $h(qG) \equiv \frac{p_i(a-c)}{b-c}$. Specifically, from Proposition 1 we have

$$\begin{aligned}h(qG) &\equiv \frac{p_i(a-c)}{b-c}\\ &= \left(\frac{a-b+qG(b-c)}{2(a-c)} + \frac{b-c}{2(a-c)}p_2^*\right)\frac{a-c}{b-c}\\ &= \frac{a-b+qG(b-c)}{2(b-c)} + \frac{1}{2}p_2^*\\ &= \frac{a-b}{2(b-c)} + \frac{qG}{2} + \frac{1}{2}p_2^*\end{aligned}$$

Then we have;

$$\frac{\partial h}{\partial(qG)} = \frac{1}{2} + \frac{1}{2}\frac{\partial p_2^*}{\partial(qG)}$$

Furthermore, from Proposition 1, we also have

$$\frac{\partial p_2^*}{\partial(qG)} = \frac{(a-c)}{(2a-c-b)} = \frac{a-c}{(a-c)+(a-b)} < 1$$

So, $\frac{\partial p_2^*}{\partial(qG)} < \frac{\partial h}{\partial(qG)}$ if

$$\frac{\partial p_2^*}{\partial(qG)} < \frac{1}{2} + \frac{1}{2}\frac{\partial p_2^*}{\partial(qG)}$$

That is, if

$$\frac{1}{2}\frac{\partial p_2^*}{\partial(qG)} < \frac{1}{2}$$

or

$$\frac{\partial p_2^*}{\partial(qG)} < 1$$

which we have already seen to be true. Therefore, it holds that the restriction is a steeper function of qG than is the optimal legal price; $\frac{\partial p_2^*}{\partial(qG)} < \frac{\partial h}{\partial(qG)}$. Given this, a necessary and sufficient condition for there to exist values of qG for which a case 2 equilibrium exists is that at $qG = 0$, it holds that $p_2^* > h$. If this were not to hold, then clearly p_2^* would be less than h for every value of qG. The value of p_2^* at $qG = 0$ is $\frac{(a-b)(2(a-c)+1)}{2(2a-c-b)}$, and the value of h at $qG = 0$ is $\frac{a-b}{2(b-c)} + \frac{1}{2}\frac{(a-b)(2(a-c)+1)}{2(2a-c-b)}$.

Thus the necessary and sufficient condition for existence of a case 2 equilibrium is

$$\frac{(a-b)(2(a-c)+1)}{2(2a-c-b)} < \frac{a-b}{2(b-c)} + \frac{1}{2}\frac{(a-b)(2(a-c)+1)}{2(2a-c-b)}$$

This reduces easily to

$$\begin{aligned} 2(a-c)(b-c)+(b-c) &< 2(a-c)+2(a-b) \\ b(2(a-c)+1+2) &< 2(a-c)(1+c)+2a+c \\ b &< \frac{2(a-c)(1+c)+2a+c}{(2(a-c)+3)} \end{aligned}$$

Appendix A-3: Proof of Proposition 3.

The equilibrium condition is $p > p_i\frac{a-c}{b-c}$. Write this as $p_i < \frac{b-c}{a-c}p$. From Proposition 1, we can see that

$$p^*_{i,2} = \frac{a-b+qG(b-c)}{2(a-c)} + \frac{b-c}{2(a-c)}p^*_2$$

Thus, the equilibrium condition is:

$$\frac{a-b+qG(b-c)}{2(a-c)} + \frac{b-c}{2(a-c)}p^*_2 < \frac{b-c}{(a-c)}p^*_2$$

This re-orders to get:

$$qG < p^*_2 - \frac{a-b}{b-c}$$

Thus, for the equilibrium of case 2 to work, the expected marginal fine must be sufficiently low, or the legal price must be sufficiently high. Alternatively, we can understand the condition as indicating that the difference between the legal price and the expected marginal fine must be greater than $\frac{a-b}{b-c}$.

Substituting into the equilibrium condition for p^*_2 from Proposition 1 gives the condition as:

$$qG < \frac{((a-b)(2(a-c)+1)+2qG(a-c)}{2(2a-c-b)} - \frac{a-b}{b-c}$$

Several tedious but straightforward steps suffice to reduce this to:

$$qG < \frac{1}{2} + (a-c) - \frac{(a-c)+(a-b)}{b-c}$$

Appendix A-4: Proof of Proposition 4.

$$\frac{\partial \pi_{l,1}}{\partial p_1} = 1 - \frac{2p_1}{a-c} - \frac{qG}{a-c} = 0$$

From which

$$p^*_1 = \frac{a-c+qG}{2}$$

This is a maximum since:

$$\frac{\partial^2 \pi_{l,1}}{\partial p_1^2} = -\frac{2}{a-c} < 0$$

Appendix B: Comparative statics

Appendix B-1: Proof of Proposition 5.

From Proposition 1:

$$\frac{\partial p_2^*}{\partial a} = \frac{\frac{\partial g}{\partial a}k - g\frac{\partial k}{\partial a}}{4((a-c)+(a-b))^2}$$

where

$$\begin{aligned} g &= (a-b)(2(a-c)+1) + 2qG(a-c) \\ \frac{\partial g}{\partial a} &= (2(a-c)+1) + 2(a-b) + 2qG \\ k &= 2((a-c)+(a-b)) \\ \frac{\partial k}{\partial a} &= 4 \end{aligned}$$

Thus, $\frac{\partial p_2^*}{\partial a} \gtreqless 0$ as $\frac{\partial g}{\partial a}k \gtreqless g\frac{\partial k}{\partial a}$. That is, as

$$\begin{aligned} & ((2(a-c)+1) + 2(a-b) + 2qG)\, 2((a-c)+(a-b)) \\ \gtreqless\ & 4\,((a-b)(2(a-c)+1) + 2qG(a-c)) \end{aligned}$$

Divide both sides by 2 to get

$$\begin{aligned} & ((2(a-c)+1) + 2(a-b) + 2qG)\, ((a-c)+(a-b)) \\ \gtreqless\ & 2\,((a-b)(2(a-c)+1) + 2qG(a-c)) \end{aligned}$$

Collect common terms and simplify;and simplify to get;

$$((a-c)-(a-b))\,(2(a-c)+1-2qG) + 2(a-b)((a-c)+(a-b)) \gtreqless 0$$

Since $2(a-b)((a-c)+(a-b)) > 0$, we have

$$((a-c)-(a-b))\,(2(a-c)+1-2qG) \geq 0 \quad \Rightarrow \quad \frac{\partial p_2^*}{\partial a} > 0$$

However, since $(a-c)-(a-b) > 0$ (from Assumption 1), a sufficient condition for $\frac{\partial p_2^*}{\partial a} > 0$ is

$$2(a-c) + 1 - 2qG \geq 0$$

That is;

$$qG \leq \frac{1}{2} + (a-c)$$

Finally, as was noted in the main text, in any case 2 equilibrium it is necessary that $qG < \frac{1}{2}$, this sufficient condition must hold, and we can conclude that $\frac{\partial p_2^*}{\partial a} > 0$.

To find the effect of b upon p_2^*, again we derive the equation in Proposition 1;

$$\frac{\partial p_2^*}{\partial b} = \frac{\frac{\partial g}{\partial b}k - g\frac{\partial k}{\partial b}}{4((a-c)+(a-b))^2}$$

where g and k are as above, and now

$$\begin{aligned} \frac{\partial g}{\partial b} &= -(2(a-c)+1) \\ \frac{\partial k}{\partial a} &= -2 \end{aligned}$$

Thus, we have $\frac{\partial p_2^*}{\partial b} \gtreqless 0$ as $\frac{\partial g}{\partial b}k \gtreqless g\frac{\partial k}{\partial b}$. That is, as

$$-(2(a-c)+1)2((a-c)+(a-b)) \gtreqless -2\left((a-b)(2(a-c)+1)+2qG(a-c)\right)$$

Divide by -2 and collect common terms;

$$(2(a-c)+1)((a-c)+(a-b)-(a-b)) \lesseqgtr 2qG(a-c)$$

That is

$$(2(a-c)+1)((a-c) \lesseqgtr 2qG(a-c)$$

Divide by $(a-c)$;

$$2(a-c)+1 \lesseqgtr 2qG$$

So the condition is

$$qG \gtreqless \frac{1}{2} + (a-c)$$

Again, we know that in any case 2 equilibrium it must hold that $qG < \frac{1}{2}$, therefore we have $qG < \frac{1}{2} + (a-c)$, and so $\frac{\partial p_2^*}{\partial b} < 0$.

Third, the effect of c upon p_2^* is

$$\frac{\partial p_2^*}{\partial c} = \frac{\frac{\partial g}{\partial c}k - g\frac{\partial k}{\partial c}}{4((a-c)+(a-b))^2}$$

where g and k are as above, and now

$$\begin{aligned} \frac{\partial g}{\partial c} &= -2(a-b) - 2qG \\ \frac{\partial k}{\partial c} &= -2 \end{aligned}$$

Thus, we have $\frac{\partial p_2^*}{\partial c} \gtreqless 0$ as $\frac{\partial g}{\partial c}k \gtreqless g\frac{\partial k}{\partial c}$. That is, as

$$(-2(a-b)-2qG)\,2((a-c)+(a-b)) \gtreqless -2\left((a-b)(2(a-c)+1)+2qG(a-c)\right)$$

Divide by -2;

$$(2(a-b)+2qG)\left((a-c)+(a-b)\right) \lesseqgtr (a-b)(2(a-c)+1)+2qG(a-c)$$

Collect common terms;

$$2qG((a-c)+(a-b)-(a-c)) \lesseqgtr (a-b)(2(a-c)+1-2((a-c)+(a-b)))$$

Simplify,

$$2qG(a-b) \lesseqgtr (a-b)(1-2(a-b))$$

and divide by $a-b$;

$$2qG \lesseqgtr 1-2(a-b)$$

That is, $\frac{\partial p_2^*}{\partial c} \gtreqless 0$ as

$$qG \lesseqgtr \frac{1}{2}-(a-b)$$

Appendix B-2: Proof of Proposition 6.

From Proposition 1;

$$\begin{aligned} p_{i,2}^* &= \frac{a-b+qG(b-c)}{2(a-c)}+\frac{b-c}{2(a-c)}p_2^* \\ &= \frac{(a-b)+(b-c)\left(qG+p_2^*\right)}{2(a-c)} \end{aligned}$$

The effect of an increase in a is

$$\frac{\partial p_{i,2}^*}{\partial a}=\frac{\left(1+(b-c)\frac{\partial p_2^*}{\partial a}\right)2(a-c)-2\left((a-b)+(b-c)\left(qG+p_2^*\right)\right)}{4(a-c)^2}$$

Thus, $\frac{\partial p_{i,2}^*}{\partial a} \gtreqless 0$ as

$$\left(1+(b-c)\frac{\partial p_2^*}{\partial a}\right)2(a-c) \gtreqless 2\left((a-b)+(b-c)\left(qG+p_2^*\right)\right)$$

Divide by 2 and simplify;

$$(a-c)-(a-b) \gtreqless (b-c)\left(qG+p_2^*-(a-c)\frac{\partial p_2^*}{\partial a}\right)$$

From Assumption 1, $(a-c)-(a-b)>0$, so we get $\frac{\partial p_{i,2}^*}{\partial a}>0$ if $qG \leq (a-c)\frac{\partial p_2^*}{\partial a}-p_2^*$.

The effect of an increase in b is

$$\frac{\partial p_{i,2}^*}{\partial b}=\frac{-1+(qG+p_2^*)+(b-c)\frac{\partial p_2^*}{\partial b}}{2(a-c)}$$

Thus, $\frac{\partial p_{i,2}^*}{\partial b} \gtreqless 0$ as

$$-1+(qG+p_2^*)+(b-c)\frac{\partial p_2^*}{\partial b} \gtreqless 0$$

That is, as

$$qG \gtreqless 1-(b-c)\,\frac{\partial p_2^*}{\partial b}-p_2^*$$

The effect of an increase in c is

$$\frac{\partial p_{i,2}^*}{\partial c}=\frac{\left(-\left(qG+p_2^*\right)+(b-c)\frac{\partial p_2^*}{\partial c}\right)2(a-c)-\left((a-b)+(b-c)\left(qG+p_2^*\right)\right)(-2)}{4(a-c)^2}$$

Thus, $\frac{\partial p_{i,2}^*}{\partial c} \gtreqless 0$ as

$$\left(-\left(qG + p_2^*\right) + (b-c)\frac{\partial p_2^*}{\partial c}\right) 2(a-c) \gtreqless \left((a-b) + (b-c)\left(qG + p_2^*\right)\right)(-2)$$

Divide by 2;

$$\left(-\left(qG + p_2^*\right) + (b-c)\frac{\partial p_2^*}{\partial c}\right)(a-c) \gtreqless -\left((a-b) + (b-c)\left(qG + p_2^*\right)\right)$$

Collect common terms;

$$(b-c)\frac{\partial p_2^*}{\partial c}(a-c) \gtreqless -(a-b) + \left(qG + p_2^*\right)\left((a-c)-(b-c)\right)$$

That is

$$(b-c)\frac{\partial p_2^*}{\partial c}(a-c) \gtreqless -(a-b) + \left(qG + p_2^*\right)(a-b)$$

or

$$(b-c)\frac{\partial p_2^*}{\partial c}(a-c) \gtreqless \left(qG + p_2^* - 1\right)(a-b)$$

Therefore, the condition is

$$\frac{\partial p_{i,2}^*}{\partial c} \gtreqless 0 \quad \text{as} \quad \frac{\partial p_2^*}{\partial c}\frac{(a-c)(b-c)}{(a-b)} \gtreqless qG + p_2^* - 1$$

Thus, if $\frac{\partial p_2^*}{\partial c} \geq 0$ the left-hand side is non-negative, and so if $qG + p_2^* - 1 < 0$ then we would have $\frac{\partial p_{i,2}^*}{\partial c} > 0$. But, from Proposition 1, p_2^* is equal to

$$\frac{1}{2}\left(\frac{(a-b)(2(a-c)+1) + 2qG(a-c)}{(a-c)+(a-b)}\right)$$

This is less than $\frac{1}{2}$ if

$$(a-b)(2(a-c)+1) + 2qG(a-c) < (a-c) + (a-b)$$

Collecting common terms this is

$$(a-c)\left(2qG-1\right) < (a-b)\left(1-(2(a-c)+1)\right)$$

i.e.,

$$2qG - 1 < -2(a-b)$$

So in the end, $p_2^* < \frac{1}{2}$ if $qG < \frac{1}{2} - (a-b)$. Notice from Proposition 5, this is the same condition for $\frac{\partial p_2^*}{\partial c} > 0$. Also, with $qG < \frac{1}{2} - (a-b)$ clearly $qG < \frac{1}{2}$, and so $qG + p_2^* - 1 < \frac{1}{2} + \frac{1}{2} - 1 = 0$. So, in short, if $qG < \frac{1}{2} - (a-b)$ we have (a) $p_2^* < \frac{1}{2}$, (b) $\frac{\partial p_2^*}{\partial c} > 0$, (c) $qG + p_2^* - 1 < 0$, and (d) $\frac{\partial p_{i,2}^*}{\partial c} > 0$.

Appendix B-3: Proof of Proposition 7.

We saw above that the intermediary's profit function in case 1 is

$$\pi_{i,1} = (1-qG)\frac{(a-c+qG)}{2(a-c)}$$

and the legal supplier's profit is

$$\pi_{l,1} = \frac{(a-c+qG)^2}{4(a-c)}$$

Thus, it is straight-forward to find that

$$\frac{\partial \pi_{l,1}}{\partial a} = \frac{2\left(a-c+qG\right)4(a-c) - 4\left(a-c+qG\right)^2}{16(a-c)^2}$$

which is positive (resp. negative, equal to zero) as

$$a-c \gtreqless qG$$

Likewise,

$$\frac{\partial \pi_{l,1}}{\partial c} = \frac{-2\left(a-c+qG\right)4(a-c) + 4\left(a-c+qG\right)^2}{16(a-c)^2}$$

which is positive (resp. negative, equal to zero) as

$$qG \gtreqless a-c$$

Appendix B-4: Proof of Proposition 8.

$$\begin{aligned} \frac{\partial \pi_{i,1}}{\partial a} &= (1-qG)\left(\frac{2(a-c)-2(a-c+qG)}{4(a-c)^2}\right) \\ &= (1-qG)\left(\frac{-qG}{2(a-c)^2}\right) \leq 0 \end{aligned}$$

Of course, for any $0 < qG < 1$, we get $\frac{\partial \pi_{i,1}}{\partial a} < 0$.

Second,

$$\begin{aligned} \frac{\partial \pi_{i,1}}{\partial c} &= (1-qG)\left(\frac{-2(a-c)+2(a-c+qG)}{4(a-c)^2}\right) \\ &= (1-qG)\left(\frac{qG}{2(a-c)^2}\right) \geq 0 \end{aligned}$$

Again, for any $0 < qG < 1$, we get $\frac{\partial \pi_{i,1}}{\partial a} > 0$.

Appendix B-5: Proof of Proposition 9.

From the equation for θ_{lr} we have

$$\frac{\partial \theta_{lr}}{\partial a} = \frac{\frac{\partial p_1^*}{\partial a}(a-c) - p_1^*}{(a-c)^2}$$

Thus $\frac{\partial \theta_{lr}}{\partial a} \gtreqless 0$ as

$$\frac{\partial p_1^*}{\partial a}(a-c) \gtreqless p_1^*$$

That is, as

$$\frac{a-c}{2} \gtreqless \frac{a-c+qG}{2}$$

Clearly, the result is $<$, so an increase in a reduces piracy in case 1.

In the same way, we get

$$\frac{\partial \theta_{lr}}{\partial c} = \frac{\frac{\partial p_1^*}{\partial c}(a-c)+p_1^*}{(a-c)^2}$$

Thus $\frac{\partial \theta_{lr}}{\partial c} \gtreqless 0$ as

$$\frac{\partial p_1^*}{\partial c}(a-c)+p_1^*$$

That is, as

$$p_1^* \gtreqless \frac{a-c}{2}$$

i.e., as

$$\frac{a-c+qG}{2} \gtreqless \frac{a-c}{2}$$

This time, clearly the result is $>$, so an increase in c increases piracy in case 1.

Appendix B-6: Proof of Proposition 10.

Using the equations in Proposition 1, straight-forward steps suffice to find that

$$p_2^* - p_{i,2}^* = \frac{(a-b)(2(a-c)+1-(a-b))+qG((a-c)+(a-b))}{2(a-c)}$$

Define

$$m = (a-b)(2(a-c)+1-(a-b))+qG((a-c)+(a-b))$$

so that;

$$\begin{aligned} \frac{\partial\left(p_2^* - p_{i,2}^*\right)}{\partial a} &= \frac{\frac{\partial m}{\partial a}2(a-c)-2m}{4(a-c)^2} \\ &= \frac{\frac{\partial m}{\partial a}(a-c)-m}{2(a-c)^2} \end{aligned}$$

Now, we have

$$\frac{\partial \theta_{lu}}{\partial a} = \frac{\frac{\partial\left(p_2^*-p_{i,2}^*\right)}{\partial a}(a-b)-(p_2^*-p_{i,2}^*)}{(a-b)^2}$$

Thus, $\frac{\partial \theta_{lu}}{\partial a} \gtreqless 0$ as $\frac{\partial\left(p_2^*-p_{i,2}^*\right)}{\partial a}(a-b) \gtreqless (p_2^*-p_{i,2}^*)$.

Making the relevant substitutions, the condition is

$$\left(\frac{\frac{\partial m}{\partial a}(a-c)-m}{2(a-c)^2}\right)(a-b) \gtreqless \frac{m}{2(a-c)}$$

That is,

$$\begin{aligned}\left(\frac{\partial m}{\partial a}(a-c)-m\right)(a-b) &\gtreqless m(a-c)\\ \frac{\partial m}{\partial a}(a-c)(a-b) &\gtreqless m((a-c)+(a-b))\\ \frac{(a-c)(a-b)}{((a-c)+(a-b))} &\gtreqless \frac{m}{\frac{\partial m}{\partial a}}\end{aligned}$$

Define

$$\frac{(a-c)(a-b)}{((a-c)+(a-b))}=\lambda<1$$

Making the relevant substitutions, the condition is

$$\lambda \gtreqless \frac{(a-b)(2(a-c)+1-(a-b))+qG((a-c)+(a-b))}{2(a-c)+1+2qG}$$

Cross multiply to get

$$\lambda\,(2(a-c)+1+2qG) \gtreqless (a-b)(2(a-c)+1-(a-b))+qG((a-c)+(a-b))$$

That is

$$(2(a-c)+1)\,(\lambda-(a-b))+qG(2\lambda-((a-c)+(a-b)) \gtreqless -(a-b)^2$$

This reduces to

$$(2(a-c)+1)\,(\lambda-(a-b))-qG(a-b) \gtreqless -(a-b)^2-qG\,(2\lambda-(a-c))$$

And finally to

$$(2(a-c)+1)\,(\lambda-(a-b)) \gtreqless (a-b)\,(qG-(a-b))-qG\,(2\lambda-(a-c))$$

Now, notice that

$$\begin{aligned}\lambda-(a-b) &= \frac{(a-c)(a-b)}{(a-c)+(a-b)}-(a-b)\\ &= \frac{(a-c)(a-b)-(a-b)((a-c)+(a-b))}{(a-c)+(a-b)}\\ &= \frac{(a-c)(a-b)-(a-b)(a-c)-(a-b)^2}{(a-c)+(a-b)}\\ &= \frac{-(a-b)^2}{(a-c)+(a-b)}\end{aligned}$$

And that

$$\begin{aligned} 2\lambda - (a-c) &= \frac{2(a-c)(a-b) - (a-c)((a-c)+(a-b))}{(a-c)+(a-b)} \\ &= \frac{(a-c)(a-b) - (a-c)^2}{(a-c)+(a-b)} \\ &= \frac{(a-c)\left((a-b)-(a-c)\right)}{(a-c)+(a-b)} \\ &= \frac{(a-c)(c-b)}{(a-c)+(a-b)} \end{aligned}$$

Therefore, the above condition is

$$\begin{aligned} &-(2(a-c)+1)\left(\frac{(a-b)^2}{(a-c)+(a-b)}\right) \\ \gtreqless\ &(a-b)\left(qG-(a-b)\right) - qG\left(\frac{(a-c)(c-b)}{(a-c)+(a-b)}\right) \end{aligned}$$

Multiply by $(a-c)+(a-b)$ to get

$$-(2(a-c)+1)(a-b)^2 \gtreqless (a-b)\left((a-c)+(a-b)\right)\left(qG-(a-b)\right) - qG(a-c)(c-b)$$

Collecting the common term qG, and carrying out a few straight-forward simplifying steps gives

$$(a-b)^2(c-b-1) \gtreqless qG\left((a-c)^2+(a-b)^2\right)$$

But the left-hand side is unambiguously negative, and the right-hand side is unambiguously positive. Thus the relevant inequality is $<$, and the result is that $\frac{\partial\theta_{lu}}{\partial a} < 0$.

Second, we have

$$\frac{\partial\theta_{lu}}{\partial b} = \frac{\frac{\partial\left(p_2^*-p_{i,2}^*\right)}{\partial b}(a-b) + \left(p_2^*-p_{i,2}^*\right)}{(a-b)^2}$$

Thus

$$\frac{\partial\theta_{lu}}{\partial b} \gtreqless 0 \quad \text{as} \quad \frac{\partial\left(p_2^*-p_{i,2}^*\right)}{\partial b}(a-b) + \left(p_2^*-p_{i,2}^*\right) \gtreqless 0$$

Recalling that

$$p_2^* - p_{i,2}^* = \frac{(a-b)(2(a-c)+1-(a-b)) + qG((a-c)+(a-b))}{2(a-c)}$$

we have

$$\begin{aligned} \frac{\partial\left(p_2^*-p_{i,2}^*\right)}{\partial b} &= \frac{-(2(a-c)+1-(a-b)) + (a-b) - qG}{2(a-c)} \\ &= \frac{2((a-b)-(a-c)) - 1 - qG}{2(a-c)} \\ &= \frac{2(c-b)-1-qG}{2(a-c)} \end{aligned}$$

Thus $\frac{\partial\left(p_2^*-p_{i,2}^*\right)}{\partial b}(a-b)+\left(p_2^*-p_{i,2}^*\right)$ is equal to

$$\frac{\left(2(c-b)-1-qG\right)(a-b)}{2(a-c)}+\frac{(a-b)(2(a-c)+1-(a-b))+qG((a-c)+(a-b))}{2(a-c)}$$

This simplifies to

$$\frac{(a-b)^2+qG(a-c)}{2(a-c)}>0$$

Thus we conclude that $\frac{\partial\theta_{lu}}{\partial b}>0$.

Finally,

$$\frac{\partial\theta_{lu}}{\partial c}=\frac{1}{(a-b)}\frac{\partial\left(p_2^*-p_{i,2}^*\right)}{\partial c}$$

But

$$\begin{aligned}\frac{\partial\left(p_2^*-p_{i,2}^*\right)}{\partial c} &= \frac{\frac{\partial m}{\partial c}2(a-c)+2m}{4(a-c)^2}\\ &= \frac{\frac{\partial m}{\partial c}(a-c)+m}{2(a-c)^2}\end{aligned}$$

where as before

$$m=(a-b)(2(a-c)+1-(a-b))+qG((a-c)+(a-b))$$

Thus the sign of $\frac{\partial\theta_{lu}}{\partial c}$ is the same as the sign of $\frac{\partial m}{\partial c}(a-c)+m$. We can calculate

$$\frac{\partial m}{\partial c}=-2(a-b)-qG$$

so that $\frac{\partial m}{\partial c}(a-c)+m$ is equal to

$$\left(-2(a-b)-qG\right)(a-c)+(a-b)(2(a-c)+1-(a-b))+qG((a-c)+(a-b))$$

This simplifies to

$$(a-b)(1-(a-b)+qG)$$

So $\frac{\partial\theta_{lu}}{\partial c}\gtreqless 0$ as $1-(a-b)+qG\gtreqless 0$. But since $(a-b)<1$, and $qG\geq 0$, we clearly have $1-(a-b)+qG>0$, and so $\frac{\partial\theta_{lu}}{\partial c}>0$.

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