Review of Economic Research on Copyright Issues, 2014, vol. 11(1), pp. 9-31

CARTEL SUSTAINABILITY AND PIRACY IN A VERTICALLY DIFFERENTIATED OLIGOPOLY

IACOPO GRASSI

Abstract. In recent years economic literature has deeply analyzed piracy and copyright violation. Nevertheless most of the contributions focus on the study of digital markets and monopoly. In this paper we concentrate on the effect the entry of a pirate may have in a vertically differentiated duopoly where originally two firms compete producing a high quality and a low quality good. We show that, under general conditions payoffs of firms might increase with piracy, since piracy may support collusion between the two firms producing the original goods and the collusive profits of the firms in presence of piracy may be bigger than the profits of Nash without piracy. This result may explain the reason why in some markets, like the fashion market, where the producers of the original brands basically control the supply chain of the sector, piracy and production of high quality fakes is huge.

1. Introduction

One of the most surprising best sellers in the Italian publishing market was the 2006 book *Gomorrah* by Roberto Saviano. This non-fiction work, placed by *The New York Times* and *The Economist* amongst the most important books of 2007, describes some of the businesses of the Camorra, a powerful Neapolitan mafia-like organization, and emphasizes the connections between criminal organizations and the productive world. The publication of this book, and its unpredictable success ended up with the author receiving death threats from the Camorra godfathers.

In the second chapter of his book, Saviano describes the typical auctions organized in the Neapolitan area by some leading Italian fashion brands. These auctions are organized in order to outsource the production:

> The auctions the big Italian brands hold in this area are strange. No one wins the contract and no one loses. The game consists in entering or not entering the race. Someone throws out an offer, stating his time and price. If his conditions are accepted, he won't be the only winner, however. His offer is like a head start the

The author wishes to acknowledge Richard Watt and one anonymous referee for their contribution, as well as the participants at the 2013 annual meeting of the European Association of Law and Economics (EALE), where a previous version of this paper was presented.

> others can try to follow. When the brokers accept a bid, the other contractors decide if they want in; whoever agrees gets the fabric. It's sent directly to the port of Naples, where the contractors pick it up. But only one of them will be paid: the one who delivers first, and with top-quality merchandise. The other players are free to keep the fabric, but they don't get a cent. The fashion houses make so much money that material isn't a loss worth considering. Even the contractors who don't satisfy the requirements of the designer labels manage to find a buyer. They sell the garments to the clans to be put on the fake-goods market. (Saviano R., *Gomorrah*, pp. 45-46)

Moreover, in Italy, seizures of large amounts of fakes and falsified items occur every day: in 2013 alone, Guardia di Finanza, the Italian law enforcement agency under the authority of the Minister of Economy and Finance, seized 22 million fakes that, according to some technicians, were indistinguishable from the original.[1]

From the point of view of an economist these anecdotal episodes are intriguing: why do big fashion brands, that control the supply chain of a sector, permit a provider to keet part of the production, which most certainly ends up being sold on the illegal fake market?

In this paper we try to shed some light on this question, contributing to the growing debate on the role of piracy in some industries, particularly relating to the fashion market.

The case of the fashion market, and the issue of whether fashion design merits extended legal protection has generated much debate among law scholars.

Raustiala and Sprigman (2006) assert that additional legal protection is unnecessary for the fashion market and that copying is beneficial to the fashion industry because of the way it speeds the fashion cycle.

A number of scholars have rejected Raustiala and Sprigman argument: Myers (2009) notes that Raustiala and Sprigman underestimate the new technologies of copying; Howard (2009) contests the sociology of the modern fashion customer used by Raustiala and Sprigman, which misunderstands the motivations of consumers using outdated assumptions about the fashion industry. Hendrick (2008) notes that fashion design protection would likely create more trouble than value because fashion design is hard to define and equally difficult to protect. Moreover if a designer were able to legally protect an article of apparel, then design pirates could easily avoid infringement by slight variations to the original design, leaving the original

[1]Source Guardia di Finanza web-site: `www.gdf.gov.it/GdF/it/Stampa/Comunicati_stampa/Comunicati_stampa_del_2014/Gennaio_2014/info-950081271.html`

designer holding virtually worthless rights for the protected design.[2] Hemphill and Suk (2008) propose that protection should be limited to very close imitations, as an intermediate stand between permitting free copying of fashion design and creating a broad right of exclusion. Harchuck (2010) underlines that it is a matter of fact that the fashion industry is losing billions of dollars every year because of piracy, and hence something needs to be done to protect these creations: fashion has become an entity, the law needs to recognize this, as it has done for so many other art forms.

On the contrary the analysis of the fashion market has been sidelined in the major debate in the economic literature even if piracy has been a widely studied phenomenon in recent years because the diffusion of Internet and broadband facilitated new forms of technological piracy and copyright violation. A survey of this literature is Peitz and Waelbroeck (2006).

Most economic analysis proposed in recent years have two assumptions in common:

- they focus on the piracy of digital products
- they concentrate on the entry of a pirate into a monopolistic market

The growth of studies on the digital piracy is undoubtedly connected to the diffusion of this kind of copyright violation, nevertheless, the absence of analysis on the fashion market, another major economic sector where diffusion of fakes and counterfeits is huge, is quite surprising.[3]

One reason may be due to the second assumption of the recent models on piracy: in the typical scheme of the works on piracy a pirate enters into a monopolistic market, producing a low quality good (the copy) that can be more or less similar to the original, thereby turning the market in a duopoly with vertical competition.[4]

Nevertheless it is hard to describe the fashion market using a monopolistic model. It is clearly an oligopoly where both horizontal and vertical differentiation exist: as in the car market, fashion brands often have an amazingly rich combination of shapes, colors, materials etc. Here we concentrate on the vertical differentiation of this sector.

Vertical differentiation occurs in a market where several goods can be ordered according to their objective quality from the highest to the lowest. In other words two goods A and B are vertically differentiated when, if the two goods have the

[2]He states: "Any benefit that could ultimately be derived from this sliver of protection is quickly negated by complications caused by trying to enforce exclusive rights against infringers. The cost of arguing whether a second design is substantially similar to the original design is significant in terms of time and money. Additionally, by the time a court reaches a final decision, the fashion design will likely no longer be in vogue." p. 272

[3]One exception is Harbi and Grolleau (2008).

[4]A relevant exception is Belleflamme and Picard (2007) who analyze piracy in an oligopolistic framework.

same price, all the consumers prefer good A to good B: in this case it is possible to say that good A is better than good B. We assume that, originally, in the fashion market there were two classes of products: high fashion goods and ordinary goods.[5] The entry of a pirate, who sells an imitation of the high fashion goods, creates a third class of goods: the fake ones.[6]

The main goal of this study is to suggest a topic that, to the best of our knowledge, has not been fully analyzed by economic theory: the effect piracy may have on the ability of the firms to collude; thus, the present article forms a part of the existing literature on the sustainability of collusion in vertical differentiated markets.

According to the results of this paper, collusion might explain the apparently unclear behavior of the fashion firms described by Saviano (2006) in his book: in a vertically differentiated duopoly, collusion may be hard but the entry of a pirate may facilitate collusion and the collusive profits of the firms in a smaller market (the one with piracy) may be greater than the Nash profits in a larger market (the one without piracy). This result confirms the intuition of some law scholars that fashion firms do not need a specific form of intellectual property protection, since in some cases piracy might benefit the producer of the original good.

The remainder of the paper is organized as follows: the next section gives an overview of the concept of collusion in oligopolies for differentiated products; the third section introduces the benchmark model without piracy; the forth section analyzes how piracy might affect collusion in a vertically differentiated duopoly; the last section draws some conclusions.

2. Collusion in differentiated oligopolies

Tacit collusion is a strategic conduct that enables firms to obtain supra-normal profits, where normal profits correspond to the equilibrium in the one shot game.[7] Tacit collusion can arise when firms interact repeatedly. They may then be able to maintain higher prices by tacitly agreeing that any deviation from the collusive path would trigger some retaliation. In order to be sustainable retaliation must be sufficiently likely and costly to outweigh the short-term benefits from cheating on the collusive path: Friedman (1971) shows that many schemes can make collusion possible.

[5]Think for example of Gucci and Zara bags.

[6]Furthermore, vertical differentiation and piracy are relevant in many markets with copyright issues: in order to avoid existing copyright legislation some firms can sell goods that are slightly different (worse) than the ones protected by law. This is not limited to the fashion market, as it may well include, for example, the toys market too or even the market for pharmaceuticals.

[7]A survey of the literature is in Feuerstein (2005).

A simple form of retaliation consists in the breakdown of collusion and the restoration of normal competitive profits: firms trust each other to maintain collusive prices, but if one of them deviates, trust vanishes and all firms start acting in their short-run interest. In such a case collusion is sustainable as an equilibrium if firms are sufficiently patient, that is if the critical value of the discount factor of the future profits δ is large enough.

Formally, let π^N, π^C ,π^D be, respectively, the per-period payoff in the Nash equilibrium of the one shot game, in collusion, and deviating from collusion by undercutting the rival. Then, for collusion to be sustainable it must be that:

$$\frac{\pi^C}{1-\delta^*} \geq \pi^D + \frac{\delta^* \pi^N}{1-\delta^*}$$

That is

$$\delta^* \geq \frac{\pi^D - \pi^C}{\pi^D - \pi^N} \equiv \sigma_i \qquad (1)$$

Thus, the higher is the value of σ_i, the more difficult it is to sustain collusion.[8] The intuition is that firms must put sufficient weight on future losses to offset the temptation of deviating.

In the presence of a differentiated product, all profits are affected by the degree of product differentiation and it is possible to analyze the relationship between product differentiation and cartel sustainability.

Debate on collusion in differentiated markets has been fruitful in the economic literature and dates back at least to Deneckere (1983).

Differentiation between brands affects the scope of collusion in two ways: firstly, it limits the short-term gains from undercutting rivals, since it becomes more difficult to attract their customers; secondly, it also limits the severity of price wars and thus the firm's ability to punish a potential deviation.

In other words, in differentiated markets, sustainability of collusion depends on two effects: the first tends to support this behavior since as differentiation decreases competition between brands increases, the second tends to encourage deviation since as differentiation decreases, gains from deviation increase.

Most of the authors involved in the debate on sustainability of collusion in differentiated markets (see, *intra alia*, Deneckere (1983), Chang (1991), Ross (1992) and Hackner (1994); more recently Lambertini (2000) and Andaluz (2010)) argue that, under some circumstances, and in particular under price competition, as products become more substitutable a deviation from collusion becomes increasingly attractive, since the second effect tends to dominate the first, and hence collusion is less

[8]For example it easy to show that, in a duopoly with linear demand, collusion is easier to sustain under price competition ($\sigma = 0.5$) than under quantity competition ($\sigma \simeq 0.53$).

stable: in other words, differentiation tends to decrease the critical value of the discount factor.

Therefore, in markets where there is close substitutability between the original goods, collusion is harder; in the rest of the paper we show how the presence of a pirate may undermine this condition, making collusion easier.

3. The Benchmark Model Without Piracy

3.1. **General Setting.** We first outline the demand side of the model build on the standard duopoly model of vertical differentiation.[9] Assume that two products of different quality, say 1 and 2, are available for consumption. Let r denote the quality of the high quality good and s denote the quality of the low quality good. As in Mussa and Rosen (1978), the consumer has the following utility function:

$$U = \text{MAX} \begin{cases} r\theta - p & \text{if she purchases the high quality good} \\ s\theta - q & \text{if she purchases the low quality good} \\ 0 & \text{if she does not purchase any good} \end{cases}$$

where p and q are the prices of the goods, and θ is a parameter describing the intensity of the preferences of the consumer for quality.

For simplicity we make the following standard assumptions:

- the quality of the high quality good is normalized to 1; $s < 1$ is the quality of the low quality good.
- the parameter θ is uniformly distributed over $[\underline{\theta}, \overline{\theta}]$, with density $\frac{1}{\overline{\theta}-\underline{\theta}}$ and $\underline{\theta} \geq 0$.

The consumer who is indifferent between the high quality and the low quality good is characterized by $\theta_i = \frac{p-q}{1-s}$, and the consumer who is indifferent between purchasing the low quality good and nothing is characterized by $\theta_j = \frac{q}{s}$. Therefore, the demands for the high quality good (D_1) and low quality good (D_2) are:

$$D_1 = \frac{1}{\overline{\theta} - \underline{\theta}} \left(\overline{\theta} - \frac{p-q}{1-s} \right) \qquad D_2 = \frac{1}{\overline{\theta} - \underline{\theta}} \left(\frac{p-q}{1-s} - \frac{q}{s} \right)$$

Following Wauthy (1996), three market configurations may arise in a standard vertical differentiation model at the price equilibrium: uncovered, covered and preempted market. In the first case we have $\frac{p-q}{1-s} > \frac{q}{s} > \underline{\theta}$; in the second $\frac{p-q}{1-s} > \underline{\theta} > \frac{q}{s}$; and in the third $\underline{\theta} > \frac{p-q}{1-s} > \frac{q}{s}$.

In what follows we concentrate on the covered market configuration with an interior solution. Moreover, without loss of generality, we assume that $\overline{\theta} = 1$ and $\underline{\theta} = 0$. In this case the demand functions become:

[9] For more details see Wauthy (1996).

$$D_1 = 1 - \frac{p-q}{1-s} \qquad D_2 = \frac{p-q}{1-s} - \frac{q}{s} \tag{2}$$

On the supply side, the marginal costs of production are assumed to be constant and, without loss of generality, we suppose they are equal to zero. This assumption seems to be consistent with all industries where the main cost is the fixed cost of developing new products or, as in the case of fashion market, new lines of clothing, footwear, accessories etc..[10] Hence in this model firms maximize their revenues.

3.2. **Price Competition.** Given the demand described in equation (2) maximizing the profits of the two firms with respect to p and q, we obtain the following reaction functions:[11]

$$p = \frac{1-s+q}{2} \qquad q = \frac{ps}{2}$$

That leads us to the following Nash equilibrium prices:

$$p^N = \frac{2(1-s)}{4-s} \qquad q^N = \frac{s(1-s)}{4-s}$$

And the following Nash equilibrium profits:

$$\pi_1^N = \frac{4(1-s)}{(4-s)^2} \qquad \pi_2^N = \frac{s(1-s)}{(4-s)^2} \tag{3}$$

3.3. **Collusion.** We now consider collusive behavior with respect to prices and given quality: we have to maximize the joint profit of the firms described by $\pi_1 + \pi_2$.

As a possible form of profit division under collusion we apply the Nash-bargaining solution, where each firm obtains profits equal to the profits derived from the non cooperative equilibrium, and the remaining surplus from cooperation is split equally between the firms; in other words each firm is assumed to get their own non-collusion Nash equilibrium profit, plus one half of the additional surplus that is gained from collusion. This assumption appears reasonable if one considers that firms can adhere to the collusive agreement on equal bases, even though their respective market shares and profits in the non cooperative setting are different.[12]

Denoting the total profit under collusion by π_{i+j}^C, the additional surplus from collusion is $\pi_{i+j}^C - \pi_i^N - \pi_j^N$. Hence under this rule the profit that each firm i obtains from collusion is given by

$$\pi_i^C = \pi_i^N + \frac{\pi_{i+j}^C - \pi_i^N - \pi_j^N}{2} = \frac{\pi_{i+j}^C + \pi_i^N - \pi_j^N}{2} \tag{4}$$

[10] Moreover in the fashion industry the copyright holder (the designer), who should receive the royalty, is often the owner of the fashion house (think of Giorgio Armani, Valentino Garavani, Marc Jacobs, etc.) and so the payments come from profit and not from royalties.

[11] For the complete computation see Appendix A.

[12] For an exhaustive treatment of the bargaining problem see Osborne and Rubinstein (1994).

In this case, maximizing aggregate profits, we have the following first-order conditions:

$$p = \frac{1 - s + 2q}{2} \qquad q = ps$$

Hence, applying the sharing rule, the collusive profits of firm 1 and firm 2 are given by:

$$\pi_1^C = \frac{8 - 5s}{8(4 - s)} \qquad \pi_2^C = \frac{3s}{8(4 - s)} \tag{5}$$

3.4. **Deviation from Collusion.**

3.4.1. *Case* 1*: firm* 1 *deviates from collusion.* In a vertically differentiated monopoly the multi-product monopolist maximizes her profits producing the high quality good only.[13]

In this case, therefore, deviating from collusion means that firm 1 keeps all the monopolistic profits for itself, not following the sharing rule described in equation (4). We have that the deviation profit for firm 1 is given by the monopolistic profit:

$$\pi_1^D = \frac{1}{4} \tag{6}$$

3.4.2. *Case* 2*: firm* 2 *deviates from collusion.* Firm 2 deviates from the collusive path fixing her best reply (given by $q = \frac{ps}{2}$) to the collusive monopolistic price set by firm 1. In this case we obtain the following deviation profits:

$$\pi_2^D = \frac{s}{16(1 - s)} \tag{7}$$

Notice that, given the asymmetry in the market share of the two firms, the deviation profits for $s \leq 0.4$ are smaller than the collusive profits described by equation (5).

Thus, it is possible to state that:

Proposition 1. If $s \leq 0.4$, i.e. if the high quality good and the low quality good are not close substitutes, firm 2 will always keep to the collusive agreement, since there is no benefit from deviation.

Proof. It follows by comparing profits of firm 2 described in equations (5) and (7): we have that $\pi_2^C \geq \pi_2^D$ if $s \leq 0.4$.

Notice that if the products are identical, so that $s = 1$, the value of expression (7) goes to infinity. This apparently unreasonable result is due to the fact that we are ignoring the constraints on the quantity produced by the firms: in our model it

[13]For an exhaustive analysis of vertically differentiated markets see Lambertini (2006).

must be that in the case of deviation of firm 2, $D_1 \geq 0$ and $D_2 \leq 1$. Applying the Kuhn-Tucker maximization conditions it is possible to show that, in this model, the latter constraint implies the former, and coincides with the non-negativity of the profits of firm 1.

In other words the deviation profits of firm 2 described in equation (7) are valid if the profits of firm 1 are at least equal to zero.

After the deviation of firm 2, the profits of firm 1 are given by $\pi_1 = \frac{2-3s}{8(1-s)}$.

In this case $\pi_1 \geq 0$ if $s \leq \frac{2}{3}$.

Thus, if $s \leq \frac{2}{3}$, i.e. if the two goods are different enough, we have to consider the interior optimum in the maximization problem, and the deviation profit is described by condition (7), if the two goods are closer substitutes ($s > \frac{2}{3}$) deviating from the collusive path firm 2 gains all the market and we have a corner solution. In this case the firm 2 price that makes $\pi_1 = 0$ is $q = s - \frac{1}{2}$ and the deviation profit is given by

$$\pi_2^D = \frac{2s-1}{4s} \tag{8}$$

3.5. **Cartel Sustainability.** Substituting the profits described by equations (3), (5), (6), (7) and (8) into equation (1), we obtain the critical discount factor of both firms in terms of s.

For firm 1 the critical discount factor is given by:

$$\sigma_1 = \frac{12-3s}{16+2s} \tag{9}$$

For firm 2 the discount factor is given by:

$$\sigma_2 = \begin{cases} \frac{5s^2-22s+8}{15s^2-24s} & \text{if } s \leq \frac{2}{3} \\ \\ \frac{7s^3-46s^2+80s-32}{12s^3-42s^2+80s-32} & \text{if } s > \frac{2}{3} \end{cases} \tag{10}$$

Since there are two asymmetric firms, the relevant discount factor, i.e. the one that ensures the sustainability of the collusive equilibrium, is given by the maximum of the those described in equations (9) and (10).

Proposition 2. If $s < \frac{4}{5}$, firm 1 decides the sustainability of the collusive agreement, and the critical discount factor is given by $\sigma_1 = \frac{12-3s}{16+2s}$; if $s > \frac{4}{5}$, firm 2 decides the sustainability of the collusive agreement, and the critical discount factor is given by $\sigma_2 = \frac{7s^3-46s^2+80s-32}{12s^3-42s^2+80s-32}$.

Proof. It follows by comparing the discount factors of the two firms described in equations (9) and (10): we have that $\sigma_1 \geq \sigma_2$ if $s \leq \frac{4}{5}$.

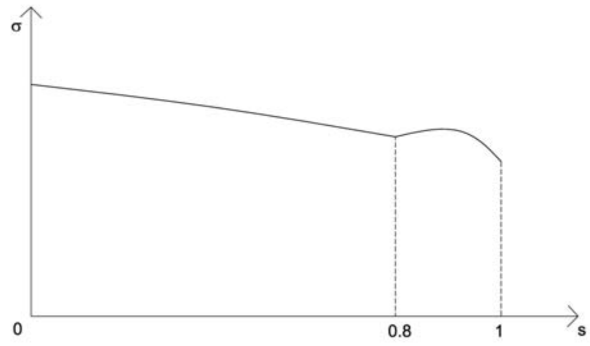

FIGURE 1. The relationship between σ and s in a vertically differentiated duopoly

Thus, the critical discount factor in a vertically differentiated duopoly is given by:

$$\sigma(s) = \begin{cases} \frac{12-3s}{16+2s} & \text{if } s \leq \frac{4}{5} \\ \\ \frac{7s^3-46s^2+80s-32}{12s^3-42s^2+80s-32} & \text{if } s > \frac{4}{5} \end{cases} \tag{11}$$

Figure 1[14]shows this equation graphically in the space s, σ.

4. THE MODEL WITH PIRACY

4.1. **General Setting.** Suppose that a pirate enters the market. Piracy consists of copyright infringement, producing a fake that is usually a low quality imitation of the high quality good, sold at a low price.

As in the previous paragraph, we consider a game where all the firms simultaneously compete à la Bertrand: firms 1 and 2 produce the two original goods (one high quality, the other low quality), the pirate produces the fake.

The presence of a pirate good modifies the utility of the consumers so that it is now given by:

$$U = \text{MAX} \begin{cases} \theta_i - p & \text{if she purchases the high quality good} \\ s\theta_i - q & \text{if she purchases the low quality good} \\ v\theta_i - w & \text{if she purchases the pirate good} \\ 0 & \text{if she does not purchase any good} \end{cases}$$

[14]The graphs in this paper were constructed using the software package Derive 6.

where the valuation of a pirate copy is equal to the valuation of an original, discounted by the factor v, where $v \leq s \leq 1$, and w is the price of the fake.

In this case we have three marginal consumers: the consumer who is indifferent between the high quality and the low quality good, $\overline{\theta} = \frac{p-q}{1-s}$; the consumer who is indifferent between the low quality good and the fake, $\underline{\theta} = \frac{q-w}{s-v}$; and the consumer indifferent between the fake and nothing, $\widetilde{\theta} = \frac{w}{v}$.

4.2. **Price competition.** In this vertical model the demand for the high quality good producer is not directly affected by the pirate, hence the reaction function of firm 1 remains the same as was obtained in the framework without piracy, i.e. $p = \frac{1-s+q}{2}$.

On the contrary, the demand for the low quality good is affected by a competitor, the pirate, and the profits of firm 2 become $\pi_2 = \left(\frac{p-q}{1-s} - \frac{q-w}{s-v}\right) q$ and her reaction function is now:

$$q = \frac{w + ps - pv - sw}{2(1-v)}$$

The profit of the pirate is described by $\pi_P = \left(\frac{q-w}{s-v} - \frac{w}{v}\right) w$, and this allows us to obtain the reaction function of firm P:

$$w = \frac{qv}{2s}$$

Given the reaction functions we can easily obtain the Nash equilibrium in the simultaneous game with piracy:[15]

$$p = \frac{1-s}{2} \frac{v - 4s + 3sv}{v - 4s + 2sv + s^2} \qquad q = \frac{s(s-v)}{s-1}(v - 4s + 2sv + s^2)$$

$$\begin{aligned} \pi_1^N &= \frac{(1-s)(6s^3v - 8s^3 + 3s^2v^2 - 14s^2v + 16s^2 + 4sv^2 - 8sv + v^2)}{4(v - 4s + 2sv + s^2)^2} \\ \pi_2^N &= \frac{(1-s)(1-v)(s-v)s^2}{(v - 4s + 2sv + s^2)^2} \end{aligned} \tag{12}$$

4.3. **Collusion, defection and critical discount factor.** As in the previous section we consider the possibility for the two firms that produce the original good to collude among themselves, and we reject the case in which they collude with the pirate.

We use the Nash Bargaining solution for the problem of the sharing of the profits of the firms described by equation (4), and we consider the possibility that either of the firms deviates from the collusive agreement.

[15]Since we are interested in analyzing the effect of piracy on collusion, we concentrate the analysis on the profits of firm 1 and firm 2.

The first relevant result we obtain is described by the following proposition:

Proposition 3. In the case of piracy, firm 2 will always keep to the collusive agreement, since the collusive profit is always bigger than the deviation profits and hence there is no incentive to deviate.
Proof. In the Appendix.

The economic intuition for this result is given by the fact that in this vertical market there is asymmetry between the firms. The main competitor of firm 2 is now the pirate, and so even when deviating from collusion, firm 2 erodes the market of the high quality producer. The gains from deviation are more than compensated by the loss (the gains for firm 2 are eroded by the pirate firm) due to the lack of compliance with the agreement with firm 1.

In the case of piracy the critical discount factor of the market coincides with the critical discount factor of firm 1 and it is possible to show[16] that it is given by the following equation:[17]

$$\sigma(s,v) = \begin{cases} \frac{D(s,v)}{E(s,v)} & \text{if } v \leq s \leq \frac{1}{4}v + \frac{1}{4}\sqrt{v(v+8)} \\ \\ \frac{F(s,v)}{G(s,v)} & \text{if } s > \frac{1}{4}v + \frac{1}{4}\sqrt{v(v+8)} \end{cases} \tag{13}$$

where the complete expressions of the functions $D(s,v)$, $E(s,v)$, $F(s,v)$, $G(s,v)$ are given in the Appendix.

In the space s, σ, equation (13) describes a family of curves, depending on v, that is possible to represent graphically (see Figure 2).

In order to analyze the effect of piracy on the collusive behavior of two firms producing different qualities of an original good, we have to compare the value of the critical discount factor expressed in equation (11) with the critical discount factor in equation (13).

To make collusion easier it must hold that the critical discount without piracy is greater than the critical discount with piracy. This happens if piracy has such an effect, which is more relevant on the Nash equilibrium profit than on collusive profit. In other words, if collusion increases the relative size of the denominator compared to the numerator, in the critical δ expressed in formula (1). With piracy, all of the profits decrease with respect to the market without piracy, but the Nash equilibrium profits should decrease by more.

[16]The complete analysis is in the Appendix B.
[17]Notice that $\frac{1}{4}v + \frac{1}{4}\sqrt{v(v+8)} \geq v$ for $0 \leq v \leq 1$.

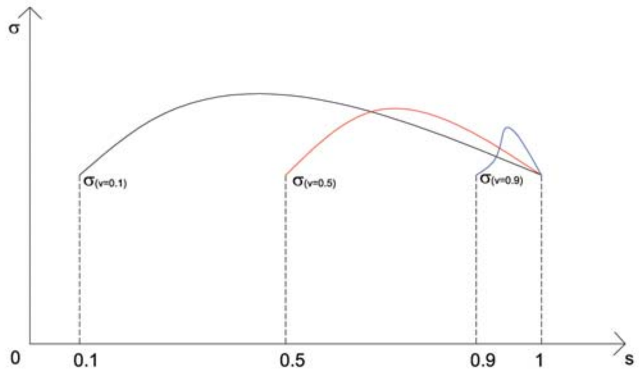

FIGURE 2. The family of curves, functions of v, that describes the relationship between σ and s in the case of piracy.

The comparison between the values of the critical discount factors is algebraically intractable, hence a numerical simulation, for low values of v, is reported in Table 1.[18]

Table 1: A numerical simulation

	σ^{NP}	$\sigma^P_{v=.1}$	$\sigma^P_{v=.2}$	$\sigma^P_{v=.3}$	$\sigma^P_{v=.4}$	$\sigma^P_{v=.5}$
$s=.2$	0.69512	0.57435				
$s=.3$	0.66867	0.59462	0.55329			
$s=.4$	0.64286	0.59102	0.58103	0.54248		
$s=.5$	0.61765	0.58168	0.58056	0.57327	0.53653	
$s=.6$	0.59302	0.56987	0.57130	0.57088	0.56401	0.53381
$s=.7$	0.56897	0.55606	0.55810	0.55999	0.56107	0.55844
$s=.8$	0.54545	0.54011	0.54198	0.54412	0.54657	0.54921
$s=.9$	0.53252	0.52162	0.5228	0.52425	0.52607	0.52846

The comparison shows us that, for most of the values of s and v, the entry of a pirate in a market for vertically differentiated goods reduces the level of the critical discount factor, making collusion easier; some values that does not respect this conditions are in bold in Table 1. For low values of v, i.e. when the fake is significantly worse than the original good, piracy makes collusion easier; this may not be true for high values of v (and consequently s), i.e. when the copy and the originals are very similar.

[18]The complete simulation is available on request from the author.

In an oligopoly the appearance of a low quality substitute as a direct and strategic effect reduces the Nash profits: to compete with the pirate, firms have to reduce their prices, and this brings about a stronger competition between them. Nevertheless such effects might not be so strong on the collusive profits.

The presence of a low quality cheaper substitute in the market attracts consumers with lower reservation prices and, to a certain extent, enlarges the market for the product. When the quality of the pirate good is high, i.e. when the original and the copy are very similar, most of the consumers shift to the pirate good, and the competition between the producers of the original goods makes defection more attractive. This may be true, in example, in markets for digital goods.

Consider now a different market, such as the fashion market. It is apparently a market where there is close substitutability between the original goods,[19] and hence, according to economic theory, collusion between the firms should be harder. Nevertheless the fashion market has a peculiarity that other markets do not have: piracy.

Moreover here the fake good is judged sensibly worse than the original. For example few elements differentiate a Gucci from a Versace bag, but a Gucci and a Versace five hundred euro bag are *both* considered very different from a twenty five euro unbranded bag.[20] In terms of our model the value of s is significantly higher than the value of v.

In this case, unlike other markets like those for digital goods, the difference between originals and copies is that the consumers more willing to spend money, i.e. with an inelastic demand, remain with the seller of original goods.

In other words, in a world with piracy the presence of copies affects the demand of original goods, and therefore the producer of such goods may be more vulnerable and inclined to collude: the drop in demand may cause a reduction in the incentive to deviate from collusion, and this effect is particularly strong when the quality of the pirate good is low.

Moreover the effects of piracy on the profits of the firms producing the high quality goods might be still more relevant. This is the case when the payoffs of the firms with piracy and collusion are greater than the payoffs in the Nash equilibrium without piracy. In such a case the firms benefit from piracy.

The profit from collusion for firm 1 in the piracy case is equal to:[21]

---

[19] We can infer this by comparing the prices of top-of-the-line products of different brands in the fashion market. For example, the latest Gucci handbags are priced similarly to comparable Prada products: both sell for over 2100 Euros. According to economic theory, this implies a high cross elasticity and hence close substitutability between the two goods. The same argument holds true for the entire product line.

[20] Following the argument expressed in the previous footnote.

[21] For the complete computation see Appendix B.

$$\pi_1^C = -\frac{A(s,v)}{8(4s^2 + 7s^2v - 16s^2 - 2sv^2 + 8sv - v^2)^2} \tag{14}$$

where

$$A(s,v) = 168s^6v - 208s^6 - 63s^5v^2 - 344s^5v + 576s^5 + 54s^4v^3 + \\ +75s^4v^2 + 96s^4v - 512s^4 + 3s^3v^4 - 14s^3v^3 - 252s^3v^2 + 512s^3v + \\ +5s^2v^4 + 72s^2v^3 - 192s^2v^2 - 6sv^4 + 32sv^3 - 2v^4$$

Comparing the profit in equation (14), with the profit in equation (3) we obtain the following result:

Proposition 4. In a vertically differentiated oligopoly, where initially two firms compete à la Bertrand producing a high quality and a low quality good, if a pirate enters the market, for the producer of the high quality good the profit from collusion with piracy is greater than the Nash equilibrium profit without piracy.
Proof. It follows from the comparison between (3) and (14).

Figure 3 describes the possible profits of firm 1 and hence illustrates Proposition 4: the dotted line represents the Nash equilibrium profit of Firm 1 without piracy (equation (3)). It is decreasing in s: if $s = 0$ Firm 1 is monopolist in the market and obtains $\pi_1 = \frac{1}{4}$; if $s = 1$ the quality of the two original goods is the same, and hence $\pi_1 = 0$ since the two firms compete à la Bertrand. The continuous curves are the family of curves representing the collusive profit of Firm 1, for different values of v: note that the relevant sections of the collusive profits, in the case of piracy, start with $s \geq v$. As v increases, the collusive profits decrease since competition with the pirate is harder, while if s increases the profit of firm 1 decreases since the competition with the other producer of the original good is more difficult.

For s sufficiently greater than v, i.e. for a low quality good that is sufficiently better than the pirate good, the profits of firm 2 increase with piracy as well.[22]

The fact that piracy induces screening between consumers, leaving the more wealthy of them in the market of the original good, allows the collusive profit to be high enough to more than compensate the loss due to the reduced demand, even in a smaller market. In such a case the profit of the pirated firms with collusion are greater than the Nash equilibrium profit without collusion.

In a vertically differentiated oligopoly piracy may facilitate the collusion, and the profits from collusion under piracy are greater than the Nash equilibrium profits

[22] A graph of this case, showing the profits of firm 2 is in Appendix C

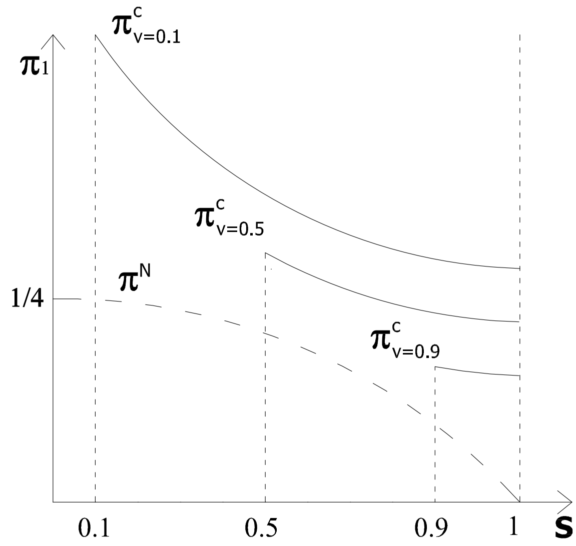

FIGURE 3. The comparison for Firm 1 between the profit of Nash without piracy and the profit of collusion with piracy, for some given value of v.

without piracy. Thus, the firms may rationally decide to favor piracy: in a world without piracy the firms would not collude, since the incentive to deviate would outweigh the benefits of collusion; on the contrary, in the case of piracy they might collude, and this could allow them to increase their final profits.[23]

This result may explain the behavior of the fashion goods brands reported in the first paragraph: the big fashion brands leave part of the production to the providers because this makes piracy easier, and in such a market firms benefit from piracy because piracy favours collusion, and the profits of collusion, even in a smaller market under piracy, may be greater than the Nash equilibrium profit in a larger market.

5. Conclusion

This paper aims to contribute to the growing debate on piracy, in particular by highlighting some aspects that the economic literature has ignored concerning the effect of piracy on sustainability of collusion in a vertically differentiated duopoly with exogenous qualities.

[23]An other interpretation is that firms producing original goods choose piracy as a signal of their willingness to collude. In this case piracy is the mechanism that is able to move the firm into the collusive equilibrium, which might insure higher profit.

The theoretical literature has pointed out a number of factors that may affect the ability of firms to collude when the explicit or implicit price fixing agreement must be self-enforcing. These factors include, for example, the number of firms, capacity constraints, demand fluctuations, multi-market contacts, etc. The piracy effect on the sustainability of collusion has been ignored, even by the recent literature on piracy.

Recently piracy has had catastrophic effects on the sales in the music market, it has changed the home entertainment market and it has played an important role in the software and video games markets. Even so, part of the literature has shown how the pirated firm can benefit from piracy. This paper situates itself in this literature.

In this work we have suggested that in certain markets, i.e. in the market where a fake is considerably worse than an original, piracy might be beneficial for the firms by supporting collusion and higher profits: the hint is that piracy may facilitate collusion between the firms producing original brands and the profit from collusion with piracy might be greater than the Nash equilibrium profit without piracy.

This does not means that piracy is beneficial in all markets. It *can* be beneficial in markets where firms compete by developing differentiated goods that are significantly better than copies. On the contrary, in markets for digital goods, where copies are basically identical to originals, the risk of annihilation of the market is very real. In such a case the pirate would be the main competitor of the producer of the high quality good and the profit of the firms would be significantly affected.

Moreover in some non-digital markets the presence of huge piracy may be interpreted as a signal of collusion between the firms producing the original goods.

Appendix A

Equilibrium in the model without piracy

Given the demand function described by equations (2), we can calculate the profits of the two firms as:

$$\pi_1 = p\left(1 - \frac{p-q}{1-s}\right); \qquad \pi_2 = q\left(\frac{p-q}{1-s} - \frac{q}{s}\right)$$

Maximizing with respect to the prices we have

$$\tfrac{\partial \pi_1}{\partial p} = 0 \Rightarrow 1 - s + q - 2p = 0$$

$$\tfrac{\partial \pi_2}{\partial q} = 0 \Rightarrow 2q - ps = 0$$

This implies:

$$p = \frac{1-s+q}{2}$$

$$q = \frac{ps}{2}$$

From the system of first-order conditions we obtain the Nash equilibrium prices $p^N = \frac{2(1-s)}{4-s}$ and $q^N = \frac{s(1-s)}{4-s}$. Substituting these into the profit of the firms, we obtain equations (3) for the Nash equilibrium profits.

$$\pi_1^N = \frac{4(1-s)}{(4-s)^2} \qquad \pi_2^N = \frac{s(1-s)}{(4-s)^2}$$

In case of collusion we have to consider the joint profits of the two firms.

$$\pi_{1+2} = p\left(1 - \frac{p-q}{1-s}\right) + q\left(\frac{p-q}{1-s} - \frac{q}{s}\right)$$

Maximizing with respect to the prices we have

$$\frac{\partial \pi_{1+2}}{\partial p} = 0 \Rightarrow 1 - s + 2q - 2p = 0$$

$$\frac{\partial \pi_{1+2}}{\partial q} = 0 \Rightarrow q - ps = 0$$

which implies:

$$p = \frac{1-s+2q}{2}$$

$$q = ps$$

From this system we obtain the collusive prices $p^C = \frac{1}{2}$ and $q^C = \frac{1}{2}s$, that lead us to the monopolistic profit $\pi_{1+2}^C = \frac{1}{4}$. Note that in a vertically differentiated monopoly the multi-product monopolist maximizes her profits by producing the high quality good only: this is a well known result in economics. In this context it means that firm 2 exits from the market. Applying the sharing rule described in the section 3.3, we obtain the collusive profits (equation (5) in the text):

$$\pi_1^C = \frac{1}{2}\left(\frac{1}{4} + \frac{4(1-s)}{(4-s)^2} - \frac{s(1-s)}{(4-s)^2}\right) = \frac{8-5s}{8(4-s)}$$

$$\pi_2^C = \frac{1}{2}\left(\frac{1}{4} + \frac{s(1-s)}{(4-s)^2} - \frac{4(1-s)}{(4-s)^2}\right) = \frac{3s}{8(4-s)}$$

We have now to consider the two possible defection cases.

The first is very simple: if firm 1 defects she keeps all the monopolistic profit, hence we have that $\pi_1^D = \pi_{1+2}^C = \frac{1}{4}$.

The optimization problem of the profit of firm 2, when she defects from the collusive agreement, can be solved using the Kuhn-Tucker method. The maximization problem is given by

$$\text{MAX } \pi_2 = q\left(\tfrac{p-q}{1-s} - \tfrac{q}{s}\right)$$
$$\text{s.t.} p = \tfrac{1}{2}$$
$$D_1 \geq 0 \Rightarrow 1 - \tfrac{p-q}{1-s} \geq 0$$
$$D_2 \leq 1 \Rightarrow \tfrac{p-q}{1-s} - \tfrac{q}{s} \leq 1$$

The Lagrangian function is:

$$L = q\left(\frac{\frac{1}{2}-q}{1-s} - \frac{q}{s}\right) + \lambda_1\left(\frac{1}{2} - s + q\right) + \lambda_2\left(q + \frac{s}{2} - s^2\right)$$

From the analysis of the four possible cases (that are $\lambda_1 = 0$, $\lambda_2 = 0$; $\lambda_1 \neq 0$, $\lambda_2 = 0$; $\lambda_1 = 0$, $\lambda_2 \neq 0$ and $\lambda_1 \neq 0$, $\lambda_2 \neq 0$) we have that if $s \leq \frac{2}{3}$ then $\pi_2^D = \frac{s}{16(1-s)}$; if $s > \frac{2}{3}$ then $\pi_2^D = \frac{2s-1}{4s}$.

Appendix B

Proof of Proposition 3 and calculation of the critical discount factor with piracy[24]

The collusive profit is given by

$$\left(1 - \frac{p-q}{1-s}\right)p + \left(\frac{p-q}{1-s} - \frac{q-w}{s-v}\right)q$$

Maximizing with respect to p and q, considering the reaction function of firm P, and applying the Nash Bargaining Solution to the sharing problem, we have the following collusive profits for the two firms:

$$\pi_1^C = -\frac{A(s,v)}{8(4s^2 + 7s^2v - 16s^2 - 2sv^2 + 8sv - v^2)^2}$$

$$\pi_2^C = -\frac{s^2 B(s,v)}{8(4s^2 + 7s^2v - 16s^2 - 2sv^2 + 8sv - v^2)^2}$$

where:

$$A(s,v) = 168s^6v - 208s^6 - 63s^5v^2 - 344s^5v + 576s^5 + 54s^4v^3 +$$
$$+75s^4v^2 + 96s^4v - 512s^4 + 3s^3v^4 - 14s^3v^3 - 252s^3v^2 + 512s^3v +$$
$$+5s^2v^4 + 72s^2v^3 - 192s^2v^2 - 6sv^4 + 32sv^3 - 2v^4$$

$$B(s,v) = -152s^4v + 176s^4 + 129s^3v^2 + 104s^3v - 320s^3 +$$
$$+18s^2v^3 - 381s^2v^2 + 480s^2v + 5sv^4 + 106sv^3 - 180sv^2 - 5v^4 + 20v^3$$

Consider now the possibility for firm 1 to deviate from the collusive path. Firm 1 fixes her best reaction $p = \frac{1-s+q}{2}$ to the collusive price of firm 2.

[24]This appendix contains some rather long-winded equations. They were arrived at using the mathematical package Mathematica 5.

In this case we have

$$\pi_1^D = \frac{(v-4s+sv+2s^2)^2}{4v^2-32sv-4sv^2+64s^2+32s^2v-64s^3}$$

This condition is valid only if the profit of the other firm is greater than 0. We have

$$\pi_2 = \frac{2s^4-3s^3v+s^2v^2-s^2v+sv^2}{(4s-v)^2(s-1)}$$

In this case $\pi_2 \geq 0$ if $s \leq \frac{1}{4}v + \frac{1}{4}\sqrt{v(v+8)}$.

Otherwise we have to consider the corner solution: firm 1 will fix the price that makes the profit of firm 2 equal to 0, given that firm 2 fixes the collusion price. In such a case we have the following deviation profits:

$$\pi_1^D = \frac{2s(2s-sv-v)}{(v-4s)^2}$$

We now turn now to the the profit of firm 2 from deviating from the collusion. In this case the two possible profits of firm 2 (interior and corner solution) are given by:

$$\pi_2^D = \frac{s^2(s-v)(1-v)}{(4s-v)^2(1-s)}$$

This is valid so long as the profit of firm 1 is greater than 0, which in this case is true if $s \geq \frac{1}{3} + \frac{1}{4}v + \frac{1}{12}\sqrt{9v^2+16}$.

Otherwise we have:

$$\pi_2^D = \frac{C(s,v)}{128s^4+72s^2v^2-8sv^3-192s^3v}$$

where

$$C(s,v) = v^3+64s^4-32s^3+2s^3v^2+5s^2v^3+40s^2v^2-16s^4v-80s^3v-6sv^3-10sv^2+32s^2v$$

Comparing the deviation profits of firm 2, with the profit that the firm obtains by colluding, we note the latter are always greater than the first, and so firm 2 always colludes (this proves Proposition 3 in the text).

This is due to the asymmetry between firm 1 and firm 2 (which produces a low quality substitute), and the fact that the pirate mainly erodes the demand of firm 2.

Hence only the critical discount factor of firm 1 is relevant. The profits we obtained allow us to calculate this value in the case of piracy as:

$$\sigma_1^C = \frac{D(s,v)}{E(s,v)} \qquad \text{if } s \leq \frac{1}{4}v + \frac{1}{4}\sqrt{v(v+8)}$$

$$\sigma_1^D = \frac{F(s,v)}{G(s,v)} \qquad \text{if } s > \frac{1}{4}v + \frac{1}{4}\sqrt{v(v+8)}$$

where

$$D(s,v) = -84sv^2 + 288s^2v + 37sv^3 + 120s^3v - 312s^4v + 120s^5v - 201s^2v^2 + \\ + 22s^2v^3 + 78s^3v^2 + 5s^3v^3 - 9s^4v^2 - 320s^3 + 368s^4 - 112s^5 - 8s^6 + 8v^3$$

$$E(s,v) = -96sv^2 + 384s^2v + 34sv^3 + 240s^3v - 552s^4v + 144s^5v - 234s^2v^2 + \\ + 28s^2v^3 + 36s^3v^2 + 2s^3v^3 + 78s^4v^2 - 512s^3 + 608s^4 - 160s^5 - 8s^6 + 8v^3$$

$$F(s,v) = -16sv^3 + 6sv^4 + 480s^4v + 104s^5v - 152s^6v + 32s^2v^2 + 8s^2v^3 - \\ - 228s^3v^2 - 5s^2v^4 + 142s^3v^3 - 363s^4v^2 - 3s^3v^4 + 10s^4v^3 + 127s^5v^2 - \\ - 320s^5 + 176s^6 + 2v^4$$

$$G(s,v) = -16sv^3 + 6sv^4 + 768s^4v + 272s^5v - 176s^6v + 32s^2v^2 + 20s^2v^3 - \\ - 336s^3v^2 - 2s^2v^4 + 184s^3v^3 - 624s^4v^2 - 6s^3v^4 + 100s^4v^3 + \\ + 64s^5v^2 - 512s^5 + 224s^6 + 2v^4$$

Appendix C

Graphical representation of the profits of firm 2

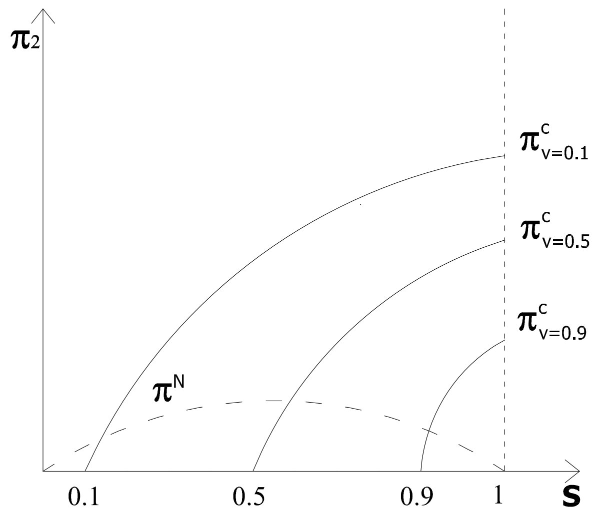

Figure 4. The comparison for Firm 2 between the profit of Nash without piracy and the profit of collusion with piracy, for some given value of v.

Figure 4 illustrates the possible profits of firm 2. The dotted line represents the Nash equilibrium profits without piracy, the continuous curves represent the collusion profits under piracy, for some values of v. We see that, as v increases the collusion profit of firm 2 decreases, while when s increases the profit of firm 2 increases, since presumably firm 2 steals some consumers from firm 1.

From the graph we note that, if s is high enough, i.e. if the two original goods are close substitutes, for firm 2 the profits from collusion under piracy can be greater than the Nash equilibrium profit without piracy.

References

Andaluz, J. (2010), "Cartel Sustainability With Vertical Product Differentiation: Price Versus Quantity Competition", *Research in Economics,* 64(4); 201-11.

Belleflamme, P. and P.M. Picard (2007), "Piracy and Competition", *Journal of Economics & Management Strategy*, 16(2); 351-83.

Chang, M.H. (1991), "The Effects of Product Differentiation on Collusive Pricing", *International Journal of Industrial Organization*, 9(3); 453-69.

Deneckere, R. (1983), "Duopoly Supergame with Product Differentiation", *Economics Letters*, 11; 37-42.

Feuerstein, S. (2005), "Collusion in Industrial Economics – A Survey", *Journal of Industry, Competition and Trade*, 5(3); 163-98.

Friedman, J.W. (1971), "A Non-Cooperative Equilibrium for Supergames", *Review of Economic Studies*, 38(113); 1-12.

Hackner, J. (1994), "Collusive Pricing in Markets for Vertically Differentiated Products", *International Journal of Industrial Organization*, 12(2); 155-77.

Harbi, S.E. and G. Grolleau (2008), "Profiting From Being Pirated by 'Pirating' the Pirates", *Kyklos*, 61(3); 385-90.

Harchuck, K.A. (2010), "Fashion Design Protection: The Eternal Plight of the Soft Sculpture", *Akron Intellectual Property Journal*, 73(4); 73-118.

Hemphill, C. and J. Suk (2008), "The Law, Culture and Economics of Fashion", *Stanford Law Review*, 61(5); 1147-200.

Hendrick, L. (2008), "Tearing Fashion Design Protection Apart at the Seams", *Washington and Lee Law Review*, 1(4); 215-73.

Howard, L. (2009), "An Uninigenious Paradox: Intellectual Property for the Fashion Design", *Columbia Journal of Law and Arts*, 32(3); 101-32.

Lambertini, L. (2000), "Technology and Cartel Stability Under Vertical Differentiation", *German Economic Review*, 1(4); 421-42.

Lambertini, L. (ed.) (2006), *The Economics of Vertically Differentiated Markets*, Cheltenham, Edward Elgar.

Mussa, M. and S. Rosen (1978), "Monopoly and Product Quality", *Journal of Economic Theory*, 18(2); 301-17.

Myers, E. (2009), "Justice in Fashion: Cheap Chic and IP Equilibrium in the United Kingdom and the United States", *American Intellectual Property Law Association Quarterly Journal*, 37(1); 47-63.

Osborne, M.J. and A. Rubinstein (1994), *A Course in Game Theory*, Cambridge MA, The MIT Press.

Peitz, M. and P. Waelbroeck (2006), "Piracy of Digital Products: A Critical Review of the Theoretical Literature", *Information Economics and Policy*, 18(4); 449-76.

Raustalia, K. and C. Sprigman (2006), "The Piracy Paradox: Innovation and Intellectual Property in Fashion Design", *University of Virginia Law Review*, 92(1); 1687-778.

Ross, T.W. (1992), "Cartel Stability and Product Differentiation", *International Journal of Industrial Organization*, 10(1); 1-13.

Saviano, R. (2006), *Gomorra*, Milan, Mondadori.

Wauthy, X. (1996), "Quality Choice in Models of Vertical Differentiation", *Journal of Industrial Economics*, 44(3); 345-53.

Department of Political Science, University of Naples Federico II - Via Rodino', Naples, Italy - tel.390812538229 - email: iagrassi@unina.it